

# Sensitivity Analysis on the Impact of Candidate Transit Projects on the Network Travel Time Evaluated by a Travel Demand Model

Camille Kamga<sup>a</sup>, Patricio Vicuna<sup>a,\*</sup>, Kyriacos Mouskos<sup>b</sup>

<sup>a</sup>University Transportation Research Center Region 2, New York City, New York, USA

<sup>b</sup>Transportation System Analyst, Nicosia, Cyprus

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## Abstract

This research aimed to address the Transit-Network Design Problem (NDP) where the Upper Level optimizes the vehicles' Network Travel Time (NTT) subject to a Budget constraint and subjects to the Lower Level that estimates the vehicles' NTT based on a Travel Demand Model (TDM). The study objective is to explore the impact of a naïve model of the more complex Transit-NDP, where stakeholders propose a priori a group of potential transit projects, including bus lines, tramways, light rail systems, and rail networks. The experimental setup involved a sensitivity analysis of budget-demand combinations for two test networks – where the Halle and the Karlsruhe test networks have 37 and 48 candidate transit projects, respectively. A complete evaluation enumeration of each budget-demand combination was conducted when it was computationally feasible, and a sampling evaluation set of 100 for the remaining evaluation test combinations. The Halle network exhibits a 47.6% NTT reduction for the base demand and a 50.8% reduction for the 160% demand level with all transit projects activated. The Karlsruhe Network exhibits an NTT reduction ranging between a base demand, resulting in a 39.7% reduction, whereas a 160% demand results in a 52.4% reduction with all transit projects activated. In both networks, a set of candidate transit projects were members of the best solution in all test runs. The sensitivity analysis demonstrated that some candidate transit projects selected at lower budget-demand combinations were not necessarily included at the best solutions of higher budget-demand levels. Overall, 18,432 test runs were conducted for the Halle network and 26,704 test runs for the Karlsruhe network, which required a total of 222 days. This analysis demonstrated the computational feasibility of conducting such a large set of experiments. In addition, the resulting dataset of the enumeration and sampling method was utilized to develop a Random Forest Regression (RFR) model, which was then utilized within a set of metaheuristics to solve this specific Transit-NDP.

**Keywords:** Transit-NDP, Travel Demand Model, Random Forest Regression Model

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## 1. Introduction

In the past few decades, urban agglomerations in the world experienced an urbanization push that resulted in the formation of large metropolitan areas – with large populations - from 3 million to over 30 million people. This trend is expected to continue as people find it more enticing to live in large urban areas instead of the traditional rural setting. It is evident that the travel time generated by the use of private vehicles to accomplish a traveler's trip is increasingly becoming longer and longer. As such, various transportation policymakers and planners are looking into ways to move people in large numbers to ease their trip-making. Therefore, the transportation system's decision-makers are tasked to offer a transportation system that will efficiently support the movement of people and goods by analyzing all potential alternatives, including transit systems. The evaluation of all potential transit-related projects requires

the solution of the corresponding travel demand model (trip generation, trip distribution, modal split, and traffic assignment) in order to identify the impact of each proposed transit skeleton line. The selection of candidate transportation network projects (e.g., roadway, transit, freight, etc.) is inherently a multi-objective one involving conflicting interests from various stakeholders such as pedestrians, micro-mobility (e.g., bicyclists, scooters, roller skaters, etc.), motorcyclists, automobile, transit (buses, BRT, LR, R), truckers, freight rail, and other.

Knowing the complexity of the transit network design problem (Transit-NDP) with multiple objectives and constraints, this study addresses the following subset problem of the Transit-NDP: Given a transportation network, identify the best transit projects – using an enumeration and sampling search strategy - that will reduce the travel time of the corresponding vehicle Network Travel Time (NTT) subject to budget constraint. This is a simplistic model of the more

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\* Corresponding author. Tel.: +1 212-650-8050;

fax: +1 212-650-8374; E-mail: [pvicuna00@citymail.cuny](mailto:pvicuna00@citymail.cuny)

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complex Transit-NDP, which is a multi-objective problem with many more restrictions. Combining the advancements in computational efficiencies with the proven effectiveness of the use of metaheuristics for solving the corresponding four-step travel demand model problem and the need to develop an evaluation tool for the selection of transit projects is the main objective of this research. The specific aim of this study is to explore the impact of various random experiments of transit project combinations by varying the corresponding demand and available budget on two test networks (Halle network and Karlsruhe network), where each candidate solution state (combination of transit projects) is evaluated utilizing a four-step travel demand model to estimate the corresponding NTT to select the best transit projects to be implemented for regional, urban, and suburban networks. In addition, this methodology aims to demonstrate that it is computationally feasible to conduct a large-scale statistical analysis on a set of candidate transit projects that could be expanded in the future to a more general set of candidate network projects (e.g., roadway, transit, environmental, etc.). Such implementation will bring consistency to the modeling – in contrast to periodic models proposed by various transport modelers - results while ensuring continuous the Transportation Network Design Problem (TNDP) updating with new data and models.

## 2. Literature Review

The TNDP falls under the category of bi-level mathematical problems and represents the main logic presented by Stackelberg [1], where the "leader" proposes a new network configuration, and the follower utilizes the resulting network configuration. The mathematical formulation of the Upper Level is as follows: Find the optimal configuration of the transportation network (best set of projects) to satisfy the stakeholders' objectives and the corresponding constraint set. The Lower Level represents the optimization of the utilization of the resulting transportation network by the various stakeholders.

An original presentation of the discrete TNDP is attributed to LeBlanc [2], where the Upper level is to find the optimal roadway projects to be built (new network state) subject to a budget constraint, and the Lower level is to estimate the resulting path/link flows (and associated travel costs (time) based on the new network configuration. The Branch and Bound method that was originally used by Leblanc was found to be rather inefficient when the number of binary variables (candidate roadway projects) increases. Later, Mouskos [3] demonstrated that the utilization of the Tabu Search metaheuristic could produce "near optimal" solutions to this specific TNDP. Subsequently, most of the publications on TNDP focused on the utilization of metaheuristics, where the solution to the UE traffic assignment became substantially faster due to the advances in the CPUs.

The Transit-NDP aims to select the set of transit routes, transit stops, transit bus and rail-car capacity and frequencies that will optimize the selected measures of performance such as network travel time/cost, OD travel time/cost, maximize the number of passengers, and other objectives as dictated by the stakeholders of the transportation network (e.g., comfort, emissions, energy consumption). Transit operators and researchers have studied different aspects of the Transit-NDP (e.g., optimizing the bus/rail frequency, optimizing the best transit routes (bus, rail, or both), optimizing the capacity of buses or car rail). Utilizing an Artificial Intelligence based model, Mahmassani and Baaj [4] defined a variation of the above Transit-NDP under which the planner seeks to determine a configuration of a set of transit routes and associated frequencies to achieve the desired objective, subject to the constraints of the problem.

Other researchers have set up the Transit-NDP as a two-objective problem, each representing the passengers and the operator [5-7]. From the user's perspective, the objective was to provide a "reasonable service" - total trip-making time, including walking to the station/destination, transit, and transfer times (converted to total trip cost) - to the passengers. In contrast, from the operators' side, the objective was entirely commercial. Ruano-Daza, Cobos, Torres-Jimenez, Mendoza and Paz [8] researched a bi-level problem to define optimal routes and frequencies for Bus Rapid Transit (BRT). This bi-level problem considers, at the upper level, the optimization of the best BRT routes and, at a lower level, the associated frequencies per BRT route to solve the problem using a metaheuristic called global-best harmony search (GHS).

The following papers refer to research conducted to produce a set of near-optimal transit routes or find near-optimal frequencies based on heuristic models. A review of the transit route network design problem was conducted by Kepaptsoglou and Karlaftis [9]. A hybrid route generation heuristic algorithm was implemented to solve the Transit-NDP problem of route generation for the design of transit networks that considered multiple characteristics such as network, frequencies, demand matrices, and parameters such as transfer time, bus seating capacity, maximum bus load factor, node sharing factor and direct capacity factor, insertion rules, and cost measures [10].

A genetic algorithm was employed to determine bus routes [11]. An Exhaustive Search algorithm, a Near-Optimal algorithm [12], and a Genetic algorithm were utilized to solve the problem of optimizing feeder bus routes considering intersection delay and irregular street networks [13]. A Simulated Annealing-based heuristic method was used to solve the Optimal Bus Transit Route network design problem considering bus stop level and aggregate zonal travel demand into a single node; a Genetic algorithm was used as a benchmark [14]. A Genetic Algorithm was proposed to optimize integrated local and express bus services. The objective function maximizes ridership (combinatorial optimization

problem), considering demand elasticity for travel time and fare, subject to minimum service frequency and fleet size constraints [15]. Ma, Hu, Chien, Liu, Yang and Ma [16] investigated the evolution assessment of urban rail transit networks; they utilized an unsupervised learning technique called principal component analysis, and a supervised learning technique called the Gaussian mixture model to divide the evolution process into stages based on temporal topological data. Breiman [17] published a research paper related to Random Forest for Regression (RFR). RFR is formed by growing trees depending on a random vector, where each random vector is identically distributed. The output values are numerical. Random forest uses the strong law of large numbers, meaning that it always converges and avoids overfitting. This supervised model can be utilized to predict the NTT based on a set of predictors.

The State Department of Transportation (DOTs) has consolidated a set of measures of effectiveness (MOEs) of transit to improve their planning, construction, and operations and in response to a greater need for accountability [18]. The MOEs of transit include Ridership Measures (ridership, passengers per capita, route percent of ridership, and boarding per day); Availability Measures (service hours, average days per week with rural transit service available, and the ratio of revenue hours to the service area population); and Internal Cost and Efficiency Measures (cost per mile, cost per rider, cost per trip, cost per service hour, and total expenses). In this research, we consider a measure of effectiveness, which is the vehicle's NTT.

**The Transit-NDP Addressed in this Study**

In this specific study, the **Upper level** is to optimize the vehicles' travel time - NTT - given a set of candidate transit projects subject to a Budget constraint, whereas the lower level represents the optimization of the trip-making of the stakeholders to achieve their activities. More specifically, it represents the traditional four-step model of trip generation, distribution, modal split, and traffic assignment [19]. Correspondingly, the lower level could have been represented in this study by an activity-based [20]. The bi-level formulation of the specific Transit-NDP addressed in this study is presented below:

$$\begin{aligned} & \text{Minimize } Z(x_a) = \sum_{a=1}^A x_a \xi_a^{\text{Car}}(x_a) \\ & \text{Subject to the Budget Constraint:} \\ & \sum_{i=1}^I C_{i,B} Y_{i,B} + \sum_{j=1}^J C_{j,T} Y_{j,T} + \sum_{k=1}^K C_{k,LR} Y_{k,LR} + \sum_{l=1}^L C_{l,R} Y_{l,R} \leq TB \end{aligned}$$

where,

- B: Bus, T: Tram, LR: Light Rail, R: Rail,
- $Y_{i,B} = \begin{cases} 1, & \text{build transit line } i \text{ as a B} \\ 0, & \text{no - built} \end{cases}$
- I: represents the maximum number of transit lines (B) that could be built,
- $i = 1, \dots, I$ : Potential number of transit lines (I) to be built,

- $C_{i,B}$ : The cost to build transit line i as B,
- $Y_{j,T} = \begin{cases} 1, & \text{build transit line } j \text{ as a T} \\ 0, & \text{no - built} \end{cases}$
- J: represents the maximum number of transit lines (T) that could be built,
- $j = 1, \dots, J$ : Potential number of transit lines (J) to be built,
- $C_{j,T}$ : The cost to build transit line j as T,
- $Y_{k,LR} = \begin{cases} 1, & \text{build transit line } k \text{ as a LR} \\ 0, & \text{no - built} \end{cases}$
- K: represents the maximum number of transit lines (LR) that could be built,
- $k = 1, \dots, K$ : Potential number of transit lines (K) to be built,
- $C_{k,LR}$ : The cost to build transit line k as LR,
- $Y_{l,R} = \begin{cases} 1, & \text{build transit line } l \text{ as a R} \\ 0, & \text{no - built} \end{cases}$
- L: represents the maximum number of transit lines (R) that could be built,
- $l = 1, \dots, L$ : Potential number of transit lines (L) to be built,
- $C_{l,R}$ : The cost to build transit line l as R,
- TB: The maximum transit budget to build a set of transit lines,
- $a = 1, \dots, A$  where A is the total number of highway links,
- $x_a$ : vehicle flow on link a,
- $\xi_a^C(x_a)$ : Generalized travel cost on the link, and it is a function ( $x_a$ , travel time, fuel consumption, tolls, etc.)

**Lower Level:**

Estimate the User Equilibrium (UE) link flows ( $x_a$ ) and corresponding travel times  $\xi_a^C(x_a)$  - needed at the Upper Level to estimate the NTT for each candidate state of the network - by solving the corresponding multimodal travel demand model (TDM). In this study, the corresponding four-step travel demand model (trip generation, trip distribution, modal split, and traffic assignment) of the commercial software VISUM was employed to estimate: a) The corresponding link flows and travel times, b) The transit/route demand assignment, per candidate state of the transportation network – the set of activated transit projects.

Hence, for each iteration of the procedure (see the general procedure in the next section), the corresponding four-step travel demand model is executed based on the set of transit projects activated. The classic four-step travel demand model applied in the two test networks considered the following VISUM models – these models can be found in the VISUM manual (PTV, 2014).

**3. Methodology**

This section outlines the approach utilized in our research with the following two main steps:

1. Conduct a statistical analysis to gain insights into the impact of various transit project combinations on traffic flow characteristics (NTT, transit demand): a) A complete enumeration of all the potential candidate transit project combinations – when computationally feasible, b) Sampling from the general population of potential transit projects where it was computationally infeasible to conduct complete enumeration,
2. Develop a Random Forest Regression (RFR) model to predict the NTT based on the dataset developed.

The RFR model developed was then utilized as a surrogate tool – instead of running the more computationally intensive TDM model - to develop a set of AI-based metaheuristic models to solve this Transit-NDP – this AI-based method can be found at [21].

The main traffic flow parameters that were estimated per set of activated transit projects are summarized in Table 1:

Table 1. Estimated Traffic Flow Parameters per Set of Activated Transit Projects.

Metric	Comments
Network Travel Time (NTT) for passenger cars	The link travel time vehicle-flows are estimated by the traffic assignment procedure of the four-step TDM; in vehicle-minutes
Mode Demand Public Transportation (PT): Bus (B), Tram (T), Light Rail (LR), Heavy Rail (R)	Passenger Car, Bicycle, B, T, LR, R, and other specific modes of transport. The modal split for all modes (e.g., car, bicycle, transit, etc.) is estimated at the second step of the four-step method; in the number of travelers per PT/vehicles
PrT	Private vehicles' total NTT; in-vehicle minutes.
PrO <sub>thero</sub>	Vehicles NTT with zero transit projects activated.
Percentage (%) impact of a set of transit projects on PrT, Red <sub>diff</sub> (%)	$(PrT_{zero} - PrT) / PrT_{zero}$ ; in %.
The maximum impact of PT on PrT (PrT <sub>max</sub> ):PrT <sub>max</sub>	PrT <sub>base</sub> - PrT <sub>all</sub> (all transit projects activated); in minutes.
Maximum percentage (%) impact of PT on PrT, (PrT <sub>max</sub> (%))	$PrT_{max} (\%) = (PrT_{zero} - PrT_{all}) / PrT_{zero}$ ; in %.
PT Share, PuT(%)	(PT travel demand per set of activated transit projects/Total travel demand) %.
Maximum PT Share: MaxPuT %	Maximum PT travel demand/Total travel demand, in %.

**General Transit-NDP Procedure (Figure 1)**

The first step involves estimating the traffic flow conditions using a chosen Travel Demand Model – in this implementation, the four-step model of the VISUM [22] TDM software, without any transit projects activated (Base Case), while assuming a fixed total travel demand. In the second step, one or more transit projects are added or removed without violating the budget constraint. The selection process is based on the chosen strategy: complete enumeration, if computationally

feasible, or random sampling - if it is computationally infeasible - of the candidate transit projects. After a new network configuration is created, the third step evaluates the latest traffic flow conditions using the corresponding updated TDM. The process returns to Step 2 to explore different combinations of transit projects. Finally, the process terminates either after a complete enumeration (if feasible) of all potential transit combinations has been explored or if a predetermined number of iterations (sampling) has been reached.

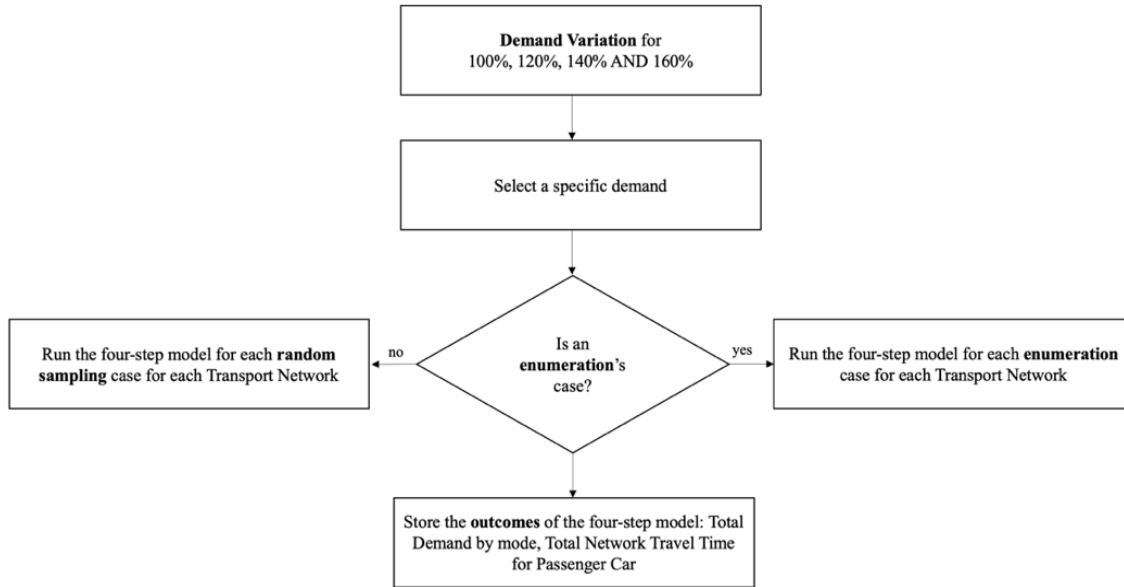


Figure 1 Flow Chart of the Enumeration and Sampling Procedure

The capital cost of each transit candidate project was estimated based on the Federal Transit Agency (FTA) Capital Cost Database 2016 version, which collects historical data on federally funded projects in the following modes: BRT, Commuter Rail, Light Rail, Heavy Rail, Tram, and Trolley [23]. Each new network configuration, due to changes in the transit network, affects the trips generated at each zone and, consequently, the resulting OD matrix, the demand at each transit route, and the routes that the travellers will follow (traffic assignment). A more general Transit-NDP model would have required to include a land-use model that captures the potential impact of new developments in the network due to the changes in the transit routes and further captures any new planned or predicted developments overall [24]. The evaluation criterion for each new transit network configuration in this study is the vehicles' NTT. The NTT was chosen in order to narrow the scope of this research, whereas recognizing that the Transit-NDP is a multi-criteria problem (e.g., various types of travel groups having different traveling criteria, economic impact overall, the economic impact of a specific area(s), travel time (cost) impact of specific routes, social impacts, environmental impacts, etc.).

The TDM that has been selected to be employed in this research, VISUM, was chosen as a convenient tool to demonstrate the methodology rather than produce real-world results. Its main advantages are that it requires a feasible computational time to be executed for the purposes of this research, it is comprehensive, capturing the full spectrum of the four-step model, and it is a well-known transportation planning software in the transportation modelling community. The four-step model TDM can be replaced by an activity-based TDM [20] or a different TDM of choice based on the availability and implementation of such models.

The Static Traffic Assignment (STA) that is embedded within VISUM could have been replaced by a Dynamic Traffic Assignment model to produce a better representation of the traffic flow conditions. The utilization of a dynamic traffic assignment model [25], while it would have improved the modelling of travel behaviour, it would have made this study computationally infeasible.

#### *Enumeration and Sampling Phase Characteristics*

##### *Enumeration Phase Characteristic*

The enumeration phase does not require any budget constraint as it conducts its experiments based on the transit project combinations. Therefore, step 2 of the general procedure presented above is only bound by the selected candidate project combinations for each network. However, the sampling phase requires that each candidate transit project combination does not violate the budget constraint. We examined two experiments involving Network 1 (Halle) and Network 2 (Karlsruhe). Network 1 comprises 37 potential transit projects, and Network 2 has 48 candidate transit projects.

**Halle Test Network Experiments (Network 1):** Explore **all candidate** project combinations of 37 candidate transit projects as Candidate transit project Combinations (37, e) per demand level, for  $e = 0, 1, 2, 35, 36, 37$ , where each element of e is the set of candidate transit projects. The **base case** is with zero candidate projects, whereas the 37 case is where all candidate projects are activated.

**Karlsruhe Test Network Experiments (Network 2):** Explore all candidate project combinations of 48 candidate transit projects: Candidate transit project Combinations (48, e) per demand level, for  $e = 0, 1, 2, 46, 47, 48$ , where each element of e is the set of

candidate transit projects. The **base case** is with zero candidate projects, whereas the 48 case is where all candidate projects are activated.

*Sampling Phase Characteristic*

**Halle Test Network Experiments (Network 1):** Conduct a random sample (size = 100) of candidate transit project combinations - Combination (37, s), for j= 3, ...,34, where s is a set of transit projects - per demand variation. **Karlsruhe Test Network Experiments (Network 2):** Conduct a random sample (size = 100) of candidate transit project combinations - Combination (48, s), for j= 3, ..., 45, where s is a set of transit projects – per demand variation.

Demand variations in both the enumeration and sampling phases are considered, including 100%, 120%, 140%, and 160%, where 100% is the base demand. Each state of the system – a network with a set of candidate

transit projects - was evaluated using the corresponding **Four-Step TDM** to estimate the related traffic flow conditions and NTT. The combination (k, n) is the number of combinations of n transit projects chosen from k transit projects - where k is greater and equal to n; for this research, we consider k=37, 48. The computational results and the corresponding analysis of the enumeration and sampling phases are presented, following with the characteristics of each test network in the next section.

*Corresponding Characteristics of the Two Test Networks*

Two test transport networks were selected - the Halle City and the Karlsruhe City, both in Germany, and each test had 37 and 48 candidate transit projects, respectively. Table 2 presents the network characteristics and main traffic flow of each test network.

Table 2. Characteristics of the VISUM Model for Halle and Karlsruhe Networks.

Characteristics	Halle Network	Karlsruhe Network
	Quantity	Quantity
<b>Base Network</b>		
Nodes	1,934	8,432
Links	4,832	23,496
Turns	13,162	72,860
Traffic Zones (TZ)	81	726
Connectors	750	5,554
OD pairs	6,561	527,076
<b>Public Transport Network</b>		
Stops	288	410
System routes	0	102
Candidate Transit Projects	37	48
No. Bus lines (B)	19	25
Tram Lines (T)	10	16
• Light Rail (LR) Lines	2	0
• Rail Lines (R)	6	7
• Time profiles	126	192
• Vehicle journeys	4377	4,388
Total Travel Demand	187,107	315,422
Traffic Assignment Model	LUCE	LUCE

It can be observed that the Karlsruhe Network has a substantially more significant number of OD pairs and travel demand than the corresponding Halle Network. As a consequence, it has a significantly higher impact on the four-step travel demand execution time in comparison to the Halle Network. The candidate transit projects for the Halle Network are Bus, Rail, Light Rail, and Tram; Bus projects (19 candidate projects): B21, B22, B24, B25, B26, B27, B28, B29, B31, B33, B34, B35, B36, B37, B38, B39, B40, B41, and B42; Tram (10 candidate projects): T1, T2, T3, T4, T5, T6, T7, T8, T9, and T10; Light Rail (2 candidate projects): S1 and SB; Rail (6 candidate projects): RB20, RB36, RB75, RB82, RE4, and RE9.

The candidate transit projects for the Karlsruhe Network are Bus, Train, and Tram; Bus (25 candidate projects): 021, 022, 023, 030, 031, 032, 042, 043, 044, 045, 046, 047, 050, 051, 052, 060, 070, 071, 073, 074, 075, 107, 123, 151, and 551; Tram (16 candidate projects): 001, 002, 003, 004, 5, 6, 7, S1, S11, S2, S3, S31, S32, S4, S41, and S5; Rail (7 candidate projects): IR, R2, R3, R5, R8, RNord, and S8.

*Corresponding Four-Step Travel Demand Model of the Two Test Networks*

The classic four-step travel demand model applied in the two test networks considered the following procedures:

- Trip generation estimates the trips generated in a Traffic Analysis Zone (TAZ) utilizing a

statistical model: trips that start and end within the TAZ, trips that start and end in other TAZs, and trips that start at other TAZs and end at the TAZ,

- Trip distribution estimates the number of trips generated from each zone to other zones, also called the Origin-Destination (OD) matrix utilizing a statistical model,
- Modal split estimates the number of trips generated from each TAZ to all other TAZs per mode of travel (passenger cars, transit (buses, light rail, rail, other), micro-mobility (bicycles, scooters, other), freight (trucks, rail), utilizing a statistical model: the mode by which to travel,
- Traffic assignment produces the routes (paths) that travelers use to travel from one TAZ to all other TAZs per mode of travel utilizing various types of models: then the analyst can estimate the corresponding link flows, link travel times, the vehicles' NTT, and other environmental measures (fuel consumption, air quality, other).

The specific models utilized in this study can be found in the manual of VISUM [22] and are not repeated here.

*Computational Results*

This section presents the results of the complete enumeration and sampling methods used to analyze the impact of various combinations of transit projects on the network travel time (NTT). The complete enumeration method is conducted for network configurations where it was computationally feasible - up to two candidate transit projects out of 37 and 48 for the Halle and Karlsruhe networks, respectively. Correspondingly, the sampling method was employed when it was computationally "infeasible" to do so within a reasonable computational time cost.

Model Runs for the two test networks:

Table 3. Model Runs for Halle Salle and Karlsruhe Network.

Network	Based demand – model Run	120% demand – model Run	140% demand – model Run	160% demand – model Run	Total
A: Halle Network	4,608	4,608	4,608	4,608	18,432
B: Karlsruhe Network	6,676	6,676	6,676	6,676	26,704
C: TOTAL (A+B)	11,284	11,284	11,284	11,284	45,136
TOTAL (C)			45,136		

Table 3 shows the model runs executed for both networks, reaching a total of 45,136 runs; 18,432 times for the Halle network and 26,704 times for the Karlsruhe network. It is noted that these test runs also depict the number of times the corresponding four-step travel demand model was executed.

*Estimation of the Computational Time (CT) to Execute the Various Enumeration and Sampling Test Runs for each Network*

The main components of the CT are the following TCT: The total computational time per test run (sec), PuT: Computational time for trip distribution for public transportation (sec), PrT: Computational time for the traffic assignment for private vehicles (sec), TPM: Computational time for the VISUM four-step Transportation Planning Model (sec), and I/O: The CT for the Input/Output (sec).

Table 4 compares two different networks, Halle and Karlsruhe, based on several CT performance metrics. For

each network, the table reports the average, standard deviation, lower 95% confidence interval, and upper 95% confidence interval for the following performance metrics: TCT/run (total computation time per test run), PuT (processing time), PrT (communication processing time), TPM (throughput), and I/O (input/output processing time). For TCT/run, the Halle network has a significantly lower average time than the Karlsruhe network (23.70 sec vs. 557.37 sec). However, the standard deviation for the Karlsruhe network is much higher (491.22 sec) compared to the Halle network (4.11 sec). For PuT, PrT, TPM, and I/O, the Halle network has lower average times compared to the Karlsruhe network. The standard deviation for PuT, PrT, and TPM is also lower for the Halle network compared to the Karlsruhe network. However, the I/O metric has a lower standard deviation for the Karlsruhe network compared to the Halle network. The Halle network generally performs better than the Karlsruhe network in terms of the reported metrics, but the Karlsruhe network has higher variability in some of the metrics.

Table 4. Computational Time per Test Run for Each Network.

Network	Metric (seconds)	TCT/run (sec)	PuT (sec)	PrT (sec)	TPM (sec)	I/O (sec)
Halle	Average	23.70	0.97	1.71	2.68	21.02
	Standard Deviation	4.11	0.81	0.45	1.17	3.25
	Lower 95% Confidence Interval	23.58	0.95	1.7	2.65	20.93
	Upper 95% Confidence Interval	23.82	0.99	1.72	2.71	21.11
Karlsruhe	Average	557.37	30.4	62.29	92.69	464.68
	Standard Deviation	491.22	45.24	68.12	86.87	426.82
	Lower 95% Confidence Interval	552.26	29.93	61.58	91.79	460.24
	Upper 95% Confidence Interval	562.48	30.87	63	93.59	469.12

**Summary of Results**

This section presents the results of the complete enumeration and sampling methods used to analyze the impact of various combinations of transit projects on the NTT.

**Impact of Public Transportation on Private Vehicles NTT (PRT)**

This section provides a summary of the enumeration and sampling test runs that were undertaken for the Halle test network (Table 5) and the Karlsruhe test network (Table 6) to estimate the impact of transit projects on the NTT.

**Halle Network Results**

Table 5. Demand – Halle Network - Impact of Public Transportation on NTT PrT.

	PT (# Trips)	CAR (PrT) (# Trips)	BIKE (# Trips)	TOTAL (# Trips)	NTT PrT (minutes)	NTT PrT (%) reduction
Base 100% Demand without PT	0	147,994	39,113	187,107	2,586,465	0.00%
Distribution	0.00%	79.10%	20.90%	100.00%		
Base 100% Demand with 37 PT projects	35,823	128,084	23,200	187,107	1,338,986	48.23%
Distribution	19.15%	68.45%	12.40%	100.00%		
Base 120% Demand without PT	0	176,796	47,732	224,529	3,254,383	0.00%
Distribution	0.00%	78.74%	21.26%	100.00%		
Base 120% Demand with 37 PT projects	43,311	152,686	28,532	224,529	1,656,597	49.10%
Distribution	19.29%	68.00%	12.71%	100.00%		
Base 140% Demand without PT	0	205,354	56,596	261,950	3,966,732	0.00%
Distribution	0.00%	78.39%	21.61%	100.00%		
Base 140% Demand with 37 PT projects	51,036	176,786	34,128	261,950	1,987,722	49.89%
Distribution	19.48%	67.49%	13.03%	100.00%		
Base 160% Demand without PT	0	233,654	65,717	299,371	4,719,464	0.00%
Distribution	0.00%	78.05%	21.95%	100.00%		
Base 160% Demand with 37 PT projects	59,011	200,372	39,989	299,371	2,303,846	51.18%
Distribution	19.71%	66.93%	13.36%	100.00%		

The Halle network exhibits an almost constant NTT reduction (demand 100% with 48.23% reduction to 180% with 53.47% reduction); in parallel, the corresponding range of the PT demand market share is (19.15% to 20.05%).

The Karlsruhe network exhibits a different behavior with an NTT reduction (demand 100% with 39.52% reduction to demand 180% with 53.99% reduction); in parallel, the corresponding range of the PT demand market share is

(27.02% to 30.60%). These results show that the Halle network is relatively uncongested, whereas the Karlsruhe network is more congested as it attracts more PT with increases in demand and exhibits a comparatively higher reduction in NTT.

**Karlsruhe Network Results**



Table 6. Demand – Karlsruhe Network - Impact of Public Transportation on NTT PrT.

	PT	CAR (PrT)	BIKE	TOTAL	NTT PrT (minutes)	NTT PrT (%) reduction
	(# Trips)	(# Trips)	(# Trips)	(# Trips)		
Base 100% Demand without PT	0	291,331	24,091	315,422	3,146,166	0.00%
Distribution	0.00%	92.36%	7.64%	100.00%		
Base 100% Demand with 48 PT projects	85,232	226,093	4,097	315,422	1,902,799	39.52%
Distribution	27.02%	71.68%	1.30%	100.00%		
Base 120% Demand without PT	0	348,561	29,899	378,460	4,251,584	0.00%
Distribution	0.00%	92.10%	7.90%	100.00%		
Base 120% Demand with 48 PT projects	106,038	267,227	5,195	378,460	2,346,669	44.80%
Distribution	28.02%	70.61%	1.37%	100.00%		
Base 140% Demand without PT	0	405,301	36,219	441,520	5,567,354	0.00%
Distribution	0.00%	91.80%	8.20%	100.00%		
Base 140% Demand with 48 PT projects	129,118	306,040	6,362	441,520	2,848,271	48.84%
Distribution	29.24%	69.32%	1.44%	100.00%		
Base 160% Demand without PT	0	461,679	43,065	504,744	7,136,023	0.00%
Distribution	0.00%	91.47%	8.53%	100.00%		
Base 160% Demand with 48 PT projects	151,663	345,263	7,818	504,744	3,393,452	52.45%
Distribution	30.05%	68.40%	1.55%	100.00%		

Demand % vs Network Travel Time for Halle Salle and Karlsruhe Network

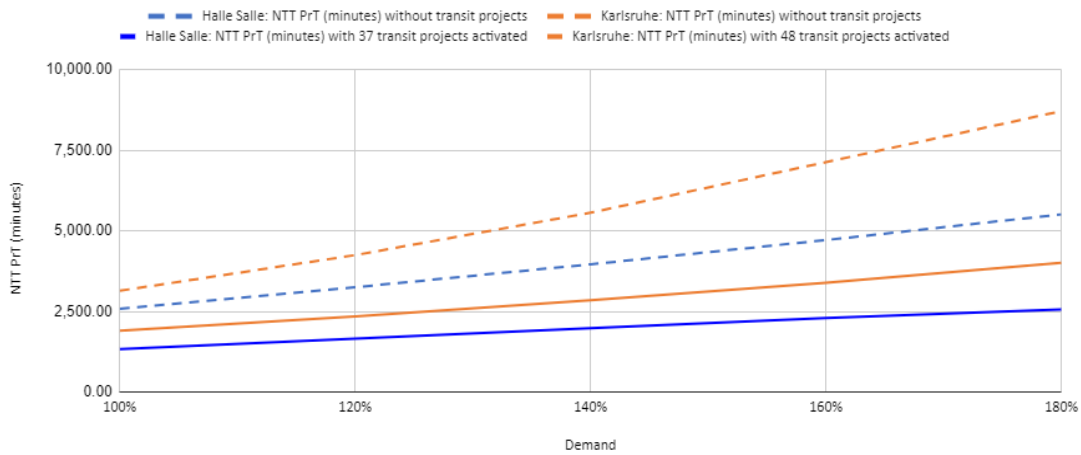


Figure 2 Demand % versus Network Travel Time PrT – For Halle and Karlsruhe Network.

axis represents the % of demand increase per test case; the y-axis represents the total Network Travel Time for PrT.

Figure 2 shows the reduction of the total Network Travel Time for PrT after adding 37 and 48 transit projects to the Halle and Karlsruhe Networks, respectively. The x-

Table 7 Demand % versus Network Travel Time PrT – For Halle and Karlsruhe Network.

Demand characteristics	Halle Network		Karlsruhe Network	
	PT (%)	NTT PrT% reduction	PT (%)	NTT PrT% reduction
Base Demand with PT	19.15%	48.23%	27.02%	39.52%
120% demand increment	19.29%	49.10%	28.02%	44.80%
140% demand increment	19.48%	49.89%	29.24%	48.84%
160% demand increment	19.71%	51.18%	30.05%	52.45%
180% demand increment	20.05%	53.47%	30.60%	53.99%

Table 7 summarizes the PT increase and NTT PrT % reduction for each demand level for the Halle and Karlsruhe Network, respectively.

- The Halle Network exhibits an almost constant NTT reduction (demand 100% with 48.23% reduction to 180% with 53.47% reduction); in parallel, the corresponding range of the PT demand market share is (19.15% to 20.05%).
- The Karlsruhe Network exhibits a different behavior with an NTT reduction (demand 100% with 39.52% reduction to demand 180% with 53.99% reduction); in parallel, the corresponding range of the PT demand market share is (27.02% to 30.60%).

The second set of experiments involved variations of both the Budget constraint (20%, 40%, 60%) and demand level (100%, 120%, 140%, 160%); see Table 8 below. The main parameters of Table 8 are:

- TRC Cost (%): percentage of the utilized budget, where 100% represents the total available budget,
- Public Transportation Performance in % (PrT %): The NTT reduction per budget-demand combination, where the basis is the estimated NTT without any transit project activated.

Table 8. Impact of Transit Project Combinations per Budget/Demand Level.

	Budget	TRC Cost (%)		PrT Performance (%)	
		Halle Network	Karlsruhe Network	Halle Network	Karlsruhe Network
Base Demand	20%	19.50%	14.60%	-45.96%	-35.10%
	40%	36.60%	37.50%	-47.04%	-38.60%
	60%	59.90%	57.30%	-47.64%	-39.70%
120% Demand	20%	17.60%	15.50%	-46.17%	-40.20%
	40%	36.00%	34.20%	-47.77%	-42.60%
	60%	52.20%	53.20%	-48.56%	-44.80%
140% Demand	20%	14.60%	19.60%	-46.43%	-44.40%
	40%	38.00%	21.60%	-49.48%	-47.90%
	60%	55.30%	51.70%	-49.64%	-49.20%
160% Demand	20%	19.90%	19.60%	-48.31%	-47.97%
	40%	30.70%	21.60%	-49.45%	-50.39%
	60%	59.00%	51.70%	-50.79%	-52.40%

Table 9. Capital Cost and NTT PrT Comparative Analysis per Demand/Budget Level.

Demand	Budget	Halle Network			Karlsruhe Network		
		20%	40%	60%	20%	40%	60%
100%	Capital Cost (MM)	\$4,506	\$8,461	\$13,849	\$6,208	\$15,967	\$24,375
100%	NTT PrT (hours)	23,295	22,831	22,571	34,009	32,186	31,623
120%	Capital Cost (MM)	\$4,056	\$8,331	\$12,067	\$6,575	\$14,546	\$22,611
120%	NTT PrT (hours)	29,196	28,327	27,903	42,354	40,676	39,102
140%	Capital Cost (MM)	\$3,377	\$8,775	\$12,778	\$8,335	\$9,179	\$21,994
140%	NTT PrT (hours)	35,417	33,400	33,294	51,604	48,298	47,094
160%	Capital Cost (MM)	\$4,608	\$7,090	\$13,646	\$8,335	\$9,179	\$21,994
160%	NTT PrT (hours)	40,662	39,760	38,709	62,252	58,957	56,599

It is observed that the Halle network exhibits an increase in transit performance from 45.96% to 50.79%, whereas the Karlsruhe network exhibits a more substantial increase from 35.10% to 52.40%. It can also be observed that most of the impact of transit projects on NTT occurs at the 20% budget level, with just a modest impact at higher budget levels. Table 9 shows a comparative analysis of the NTT per budget/demand level and the

associated capital cost. It is observed that for the Halle and Karlsruhe networks, the 100% Budget level exhibited a higher NTT PrT than the corresponding 60% Budget level in all demand variations; however, it exhibited a lower NTT in all other budget levels. Table 10 below shows a summary of the results for the best solutions found, including the corresponding NTT and capital cost spent per budget-demand combination.

Table 10. Best Transit Project Combinations per Budget Level for 100%, 120%, 140%, and 160% Demand.

Budget	Base Demand: TRC Combinations		120% Demand: TRC Combinations		140% Demand: TRC Combinations		160% Demand: TRC Combinations	
	Halle Network	Karlsruhe Network	Halle Network	Karlsruhe Network	Halle Network	Karlsruhe Network	Halle Network	Karlsruhe Network
20%	19 projects selected	21 projects selected	18 projects selected	24 projects selected	14 projects selected	21 projects selected	16 projects selected	21 projects selected
	Demand PrT:29,094	Demand PrT:70,151	Demand PrT:32,130	Demand PrT:89,268	Demand PrT:37,198	Demand PrT:100,182	Demand PrT:47,180	Demand PrT:117,737
	NTT PrT: 1,397,711	NTT PrT: 2,040,557	NTT PrT: 1,751,740	NTT PrT: 2,541,217	NTT PrT: 2,125,036	NTT PrT: 3,096,210	NTT PrT: 2,439,709	NTT PrT: 3,735,124
	Capital Cost: \$4,506 MM	Capital Cost: \$6,208 MM	Capital Cost: \$4,056 MM	Capital Cost: \$6,575 MM	Capital Cost: \$3,377 MM	Capital Cost: \$8,335 MM	Capital Cost: \$4,608 MM	Capital Cost: \$8,335 MM
40%	27 projects selected	34 projects selected	24 projects selected	35 projects selected	29 projects selected	31 projects selected	25 projects selected	31 projects selected
	Demand PrT:31,094	Demand PrT:80,702	Demand PrT:37,217	Demand PrT:95,489	Demand PrT:48,738	Demand PrT:124,109	Demand PrT:52,813	Demand PrT:145,440
	NTT PrT: 1,369,849	NTT PrT: 1,931,131	NTT PrT: 1,699,643	NTT PrT: 2,440,537	NTT PrT: 2,003,996	NTT PrT: 2,897,901	NTT PrT: 2,385,615	NTT PrT: 3,537,425
	Capital Cost: \$8,461 MM	Capital Cost: \$15,967 MM	Capital Cost: \$8,331 MM	Capital Cost: \$14,546 MM	Capital Cost: \$8,775 MM	Capital Cost: \$9,179 MM	Capital Cost: \$7,090 MM	Capital Cost: \$9,179 MM
60%	29 projects selected	37 projects selected	31 projects selected	42 projects selected	32 projects selected	37 projects selected	31 projects selected	37 projects selected
	Demand PrT:33,737	Demand PrT:81,903	Demand PrT:41,005	Demand PrT:103,221	Demand PrT:49,648	Demand PrT:127,592	Demand PrT:57,107	Demand PrT:148,532
	NTT PrT: 1,354,245	NTT PrT: 1,897,383	NTT PrT: 1,674,198	NTT PrT: 2,346,115	NTT PrT: 1,997,634	NTT PrT: 2,825,665	NTT PrT: 2,322,569	NTT PrT: 3,395,919
	Capital Cost: \$13,849 MM	Capital Cost: \$24,375 MM	Capital Cost: \$12,067 MM	Capital Cost: \$22,611 MM	Capital Cost: \$12,778 MM	Capital Cost: \$21,994 MM	Capital Cost: \$13,646 MM	Capital Cost: \$21,994 MM

123, 151, 551, R3, R5, R8, S1, S11, S2, S3, S31, S4, and S5.

Sample best solutions: Halle network

- **Lower-level demand:** Base demand, budget 20%, and 19 projects selected:
  - B21, B22, B24, B25, B26, B27, B28, B29, B35, B36, B40, SB, T1, T2, T3, T4, T5, T6, and T8.
- **Higher-level demand:** 160% demand, budget 20%, and 16 projects selected:
  - B21, B24, B26, B27, B31, B34, B37, B38, SB, T1, T2, T4, T6, T7, T8, and T10.
- The transit projects B22, B25, B28, B29, B35, B36, B40, T3, and T5 did not appear at higher-level.

- The transit projects 021 and 075 did not appear at higher-level.

As is demonstrated above, a candidate transit project may be selected at a lower budget or demand level, but it may not be selected at a higher budget or demand level. This result demonstrates the importance of such sensitivity analysis that may aid the decision-makers in the selection of the best transit projects.

This analysis concentrated on the impact of transit projects on the transportation network NTT through the conduct of a set of experiments by varying the demand and the budget allocation. It demonstrates that it would be best to examine various levels of budget and demand levels such that the decision-makers can make better and more informed decisions. It is again stated that the NTT is only one decision parameter in the evaluation of new transit projects as the Transit NDP is a multi-objective problem involving different stakeholders having varying objectives and constraints.

### Random Forest Regression (RFR) Model

Random Forest Regression is a technique that employs an ensemble of decision trees. Each tree in the ensemble

Sample best solutions: Karlsruhe network

- **Lower-level demand:** Base demand, budget 20%, and 21 projects selected:
  - 001, 003, 004, 005, 006, 008, 021, 030, 031, 050, 060, 070, 071, 074, 075, 123, 151, 551, R8, S11, and S5.
- **Higher-level demand:** 160% demand, budget 60%, and 37 projects selected:
  - 001, 002, 003, 004, 005, 006, 008, 022, 023, 030, 031, 032, 042, 043, 044, 045, 047, 050, 051, 060, 070, 071, 073, 074,

relies on a random vector of values independently sampled and shares a consistent distribution across all trees within the forest. This collective approach allows Random Forest Regression to make accurate predictions for continuous numeric outcomes by leveraging the diverse insights of individual trees [17].

The dataset that was developed under enumeration and sampling was utilized to develop the RFR method to produce estimates of the corresponding NTT for each test network. Based on the dataset, we split the data by 80% for training and 20% for testing; an RFR Model (RFRM) was trained to forecast the NTT PrT with 2,000 decision trees with a test accuracy close to 90%. This supervised machine learning model was then utilized to develop hybrid AI-based metaheuristic models utilizing simulated Annealing and Tabu Search [21, 26, 27].

#### 4. Conclusions

The aim of the research was to address a problem of the Transit-NDP, which involves identifying the optimal transit projects for reducing the Network Travel Time (NTT) of vehicles given a fixed budget and transportation network. The research considers a simplistic model of the more complex Transit-NDP, a multi-objective problem involving many stakeholders with often conflicting objectives with many more constraints. This specific Transit-NDP assumes that the stakeholders propose a group of potential transit projects, which may include bus lines, tramways, light rail systems, and rail networks. In the future, a set of candidate transit projects could be identified through one or more models, and another set to be proposed by the various stakeholders.

The Transit-NDP addressed in this study belongs to the broad class of bi-level formulation: The Upper level optimizes the vehicles' NTT subject to a Budget constraint based on the activated transit projects; the lower level utilizes a TDM that estimates the trips (person-demand) generated per traffic zone (trip generation - step 1), the number of trips per OD pair (trip distribution - step 2), the number of trips per transport mode (modal split - step 3) - the transportation planning software utilized in the study also estimates the trips for each transit route, the path and link flow for vehicles (e.g., passenger cars, trucks) and the corresponding link travel times (traffic assignment - step 4). The vehicle - the model utilizes an average vehicle occupancy to convert person trips to vehicle trips - link flows and travel times are then used to estimate the NTT for the Upper level. The estimation of the vehicles' NTT could be used by policymakers in justifying subsidies for transit projects.

The dataset developed under this statistical analysis was then utilized to develop the corresponding RFR model to produce an estimate of the NTT, with an average 90% success rate. The RFR model was then utilized to develop hybrid AI-based models utilizing the metaheuristics Simulated Annealing, and Tabu Search to

produce "near optimal" solutions to the specific Transit-NDP addressed in this study, which is reported in the Vicuna (2022) dissertation thesis (see [21]).

The main outcomes of the budget-demand sensitivity analysis are: i) The Halle network exhibits an almost constant NTT reduction ranging between (100%, demand 48.23% reduction to 180% demand, 53.47% reduction); in parallel, the corresponding range of the PT demand is (19.15% to 20.05%). The Karlsruhe network exhibits a different behavior with an NTT reduction ranging between - a 100% demand results in a 39.52% reduction, whereas a 180% demand results in a 53.99% reduction); in parallel, the corresponding range of the PT demand % is 27.02% to 30.60%, respectively; ii) Impact of Budget increase on NTT for Halle network: It can be observed that the most substantial reduction in NTT occurs with the budget level at 20% ranging from 45.96% to 49.38% (range difference of 3.42%) for the corresponding demand changes from 100% to 180%, whereas the corresponding NTT decrease at a budget level of 80% exhibits only a smaller decrease of 2.31% to 4.08% above the corresponding decrease of the 20% budget level; iii) Impact of Budget increase on NTT for the Karlsruhe network: It can be observed that the most substantial reduction in NTT occurs with the budget level at 20% ranging from 35.14% to 49.83% (14.69% range difference) for the corresponding demand changes from 100% to 180%, whereas the corresponding NTT decrease at a budget level of 80% exhibits a smaller decrease of 4.16% to 5.72% above the corresponding decrease of the 20% budget level; iv) The following observations applies to both test networks: a) it is not necessary that a project that appears at the best solution of a lower budget level also appear at the best solution of a higher budget level, b) it is not necessary for a project that appears in the best solution of a lower demand level to also appear at the best solution of a higher demand level, c) a set of candidate transit projects were members of the best solution of all test runs. The decision-makers could then use these sensitivity analysis results in constructing the corresponding transit project list; v) The Total Computational (TCT) for executing all the test runs for the Halle network for all scenarios is 151.68 hours or 6.3 days (23,040 runs). The TCT for executing all the test runs for the Karlsruhe network for all scenarios is 5,168 hours or 215.3 days (33,380 runs). These execution times could be substantially lowered with the utilization of a more powerful server and/or the utilization of parallel computing. In addition, the TDM software code could be used directly as a computer code rather than through running the software.

The principal impact of this analysis is that such a sensitivity analysis on budget-demand combinations could systematically aid in the decision-making process. A better transit project combination may be obtained by examining the impact of projected higher travel demand and budget levels - e.g., potentially requesting more funds to achieve better network results. In addition, the overall reduction in NTT for vehicles could be used as a tool to estimate and justify the subsidies for

implementing such transit projects. It is emphasized that the impact on NTT is just one parameter that the decision makers need to take into consideration among the multiple parameters that may be considered among the various types of stakeholders (pedestrians, micro-mobility (e.g., bicycles, scooters), transit (e.g., bus, Tram, LR, R), taxis, trucks). Decision-makers can identify a set of potential candidate transit projects through studies/workshops with different stakeholders, city planners, and transportation planners to include them in a transportation network.

A major outcome of this research is that it was demonstrated that it is computationally feasible to conduct a model-based statistical analysis of the impact of candidate transit projects. It also sets the framework to develop Artificial intelligence based on metaheuristic models, which is demonstrated in the second part of the study that utilized these results to develop a set of metaheuristics to solve the Transit-NDP [21].

### 5. Future Work

A real-life implementation of the methodology followed would need to establish a continuously calibrated TDM of choice, a method to define (with models or studies, or both) all candidate network projects – transit, roadway, other – and the corresponding computational environment, such that all stakeholders will have a tool to evaluate one or more candidate projects. Its implementation will bring consistency to the modeling – in contrast to periodic models proposed by various transport modelers - results while ensuring a continuous updating of new data and models. A procedure could be undertaken to gather traffic flow data from various installed roadway sensors, vehicle location systems (e.g., GPS), socioeconomic data, travelers' surveys, and candidate transit costs to establish a self-calibrated model. The model could be extended to include a travel cost function instead of travel time, change the objective function to two objectives to include also the transit demand, incorporate an activity-based TDM, and one or more models to find various candidate transit projects (mode, route, frequencies, capacity) in addition to transit projects proposed by various stakeholders, incorporate roadway projects as well into the set of candidate transit projects, other. Furthermore, particular focus should be made on designing and implementing the best computing platform in terms of hardware and software architecture to improve the computational efficiency of the whole system.

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