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Equivalent Lumped Criterion for Unsteady Heat Conduction in a Vertical Planar Wall with Natural Convection to a Nearby Quiescent Fluid

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Abstract

Within the framework of the potent lumped model, unsteady heat conduction takes place in a solid body where the mean temperature varies with time. Conceptually, the lumped model subscribes to the notion that the external convective resistance at the body surface dominates the internal conductive resistance inside the solid body. For forced convection heat exchange between a solid body and a fluid, the lumped model criterion entails to the lumped Biot number, $Bi_l < 0.1$, in which the mean convective coefficient depends on the impressed fluid velocity. In contrast, for natural convection heat exchange between a solid body and a fluid, the mean convective coefficient depends on the solid-to-fluid temperature difference. As a consequence, the lumped Biot number criterion must be modified to read $Bi_l <$ 0.1, wherein the maximum mean convective coefficient occurs at the initial temperature T_{in} and time t_{in} for cooling or at a future temperature T_{fut} and time t_{fut} for heating. In this paper, the equivalence of the lumped Biot number criterion for a vertical planar wall is deduced employing the thermal conductivity of the solid and the initial or future Rayleigh number as the deciding factors..

Keywords: *vertical planar wall, natural convection, mean convective coefficient, nearby fluid, nonlinear lumped equation, lumped Biot number criterion*

1. Introduction

When a solid body is immersed in an extensive fluid at a different temperature, heat exchange between the body surface and the fluid can occur by either forced convection or natural convection in most situations. When a solid body is absent of internal heat generation, the heat exchange with the fluid is dependent upon two resistances: (1) the internal conductive resistance inside the solid body and (2) the external convective resistance at the interface between the solid body surface and the fluid. In this regard, three cases of importance can be categorized. One limiting case deals with negligible internal conductive resistance, another limiting case deals with negligible external convective resistance and an intermediate case where the internal conductive resistance is comparable to the external convective resistance as cited in References [1-5]. Focusing on the first limiting case, it is associated with a small temperature difference between the center and surface of the solid body and a large temperature difference between the surface of the solid body and the fluid in contact. Physically, this case signifies that during a cooling (heating) period, the solid body can be considered as a "lump" possessing nearly uniform temperature at any instant of time; this is synonymous with a zero dimension body. In other words, unsteady heat conduction takes place in a quasi–isothermal solid body whose mean temperature changes with time only. This is the rationale that sets the groundwork for the assumption underlying the lumped model in unsteady heat conduction.

The applicability of the lumped model for unsteady heat conduction in a solid body is tied up to a criterion expressible by the lumped Biot number $Bi_l < 0.1$ as cited in [1-5]. However, these textbooks habitually omit that the heat exchange between the solid body and the fluid has to be ensued by forced convection**.** Since forced convection is a linear mode of heat transfer (generally impervious to temperature changes), the lumped $Bi_l < 0.1$ rests on a mean convective coefficient which remains constant during the cooling (or heating) period. In engineering practice, the lumped model simplification is normally satisfied by small solid bodies of regular or irregular size, and/or solids with large thermal conductivity, and/or adjacent fluids with weak mean convective coefficients [1-5]. Regardless of the prevalent heat transfer mode, either forced or natural convection, the mean convective coefficient $\bar{h}$ is usually evaluated from mean Nusselt number correlation equations reported in [1-5].

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On the contrary, when natural convection cools (heats) solid bodies, natural convection being a nonlinear mode of heat transfer, possesses a mean convective coefficient that is not constant, but depends on the instantaneous mean temperature, which in turn depends on time. The standard lumped criterion needs to be modified and, in consequence has to be rewritten as $Bi_l < 0.1$. Under these circumstances, there are two possible scenarios: cooling and heating. For cooling, the maximum mean convective coefficient, which is associated with the initial temperature T_i, at an initial time t_i, whereas for heating, the maximum mean convective coefficient has the same significance, but is connected to a future temperature T_{fut} at a future time t_{fut}.
This technical note seeks to establish an equivalent criterion for the applicability of the lumped model embodying nonlinear natural convection that bypasses the lumped Biot number $Bi_l < 0.1$. As a proof-of-concept, a heated vertical planar wall dissipating heat to a quiescent fluid, such as air, water and engine oil is taken. The alternative criterion uses: a) the thermal conductivity of the solid k_s, b) the slenderness ratio of the planar wall H/L and c) the initial Rayleigh number based on the initial temperature $Ra_{H,in}$ as the main ingredients. This approach means that the mean convective coefficient □ does not need to be evaluated with a Nusselt number correlation up front..

2. Natural Convection Mode

Dimensional analysis for natural convection heat transfer between a heated solid body and a fluid at rest articulates the mean Nusselt number $\overline{Nu}_{Lc}$ with the Rayleigh number Ra_{Lc} and the Prandtl number Pr by way of the double–valued function (Jaluria [6]:

$$\overline{Nu}_{Lc} = f\,(\underbrace{Ra_{Lc}}_{\substack{primary\\ variable}},\ \underbrace{\Pr}_{\substack{secondary\\ variable}}) \tag{1}$$

where L_c stands for the characteristic length of the solid body.

3. Lumped Heat Equation

The lumped heat equation for a solid body with uniform initial temperature T_{in} exchanging heat with a stagnant fluid at a low free-stream temperature T_∞ is

$$\rho c_v V \frac{d\bar{T}}{dt} = -\bar{h}A\,(\bar{T} - T_\infty), \quad \bar{T} = T_{in},\ t = 0 \tag{2}$$

where the participating symbols are described in the Nomenclature. The validity of eq. (2), also named the zero dimensional heat equation, requires that the lumped Biot number [1-5],

$$Bi_l = \frac{\bar{h}}{k_s}\left(\frac{V}{A}\right) < 0.1 \tag{2a}$$

Actually, the lumped Biot number carries uncertainties, because of the uncertainties in the experimental determination of the mean convective coefficient $\bar{h}$ usually comprise error bands from $\pm 10\%$ to $\pm 20\%$ (Holman [4]). As a result, the threshold value of 0.1 is not that rigid and may range between a low 0.083 and a high 0.125.

When the solid body is a planar wall, the volume–to–surface area ratio $\frac{V}{A}$ is the semi-thickness L, so that eq. (2) recedes to

$$\rho c_v L \frac{d\bar{T}}{dt} = -\bar{h}(\bar{T} - T_\infty), \quad \bar{T} = T_{in},\ t = 0 \tag{3}$$

and the lumped Biot number becomes

$$Bi_l = \frac{\bar{h}L}{k_s} < 0.1 \tag{3a}$$

4. Correlation Equations

The mean convective coefficient $\bar{h}$ for natural convection between a vertical planar wall and a still fluid is obtained from the pair of correlation equations for the mean Nusselt number reported by McAdams [7]:

For laminar regime:

$$\overline{Nu}_H = 0.59\,Ra_H^{1/4} \quad \text{with } 10^4 < Ra_H < 10^9 \text{ and } Pr > 0.7 \tag{4a}$$

For turbulent regime:

$$\overline{Nu}_H = 0.10\,Ra_H^{1/3} \quad \text{with } 10^9 < Ra_H < 10^{13} \text{ and } Pr > 0.7 \tag{4b}$$

where the intervening thermophysical properties of the fluid are evaluated at the film temperature $T_f = \frac{1}{2}(T_s + T_\infty)$.

Incidentally, there is a good matching in eqs. (4a) and (4b) because when the common $Ra_H = 10^9$, $\overline{Nu}_H = 105$ from eq. (4a) and 100 from eq. (4b).

Isolating the mean convective coefficient $\bar{h}$ in eqs. (4a) and (4b) gives way to the equations

$$\bar{h} = \frac{k_f}{H}\left(0.59\,Ra_H^{1/4}\right) \quad \text{for laminar regime} \tag{5a}$$

$$\bar{h} = \frac{k_f}{H}\left(0.10\,Ra_H^{1/3}\right) \quad \text{for turbulent regime} \tag{5b}$$

Subsequently, eqs. (5a) and (5b) are conveniently rewritten in terms of primitive geometric and thermal quantities as follows

$$\bar{h} = \frac{0.59}{H^{1/4}}\left[k_f\left(\frac{g\,\beta}{\alpha\nu}\right)^{1/4}\right](\bar{T} - T_\infty)^{1/4} \quad \text{for laminar regime} \tag{6a}$$

$$\bar{h} = 0.10\left[k_f\left(\frac{g\,\beta}{\alpha\nu}\right)^{1/3}\right](\bar{T} - T_\infty)^{1/3} \quad \text{for turbulent regime} \tag{6b}$$

The pair of nonlinear equations (6a) and (6b) establishes single-valued functions of the dependent variable $\bar{h}$ in terms of the independent variable $\bar{T} - T_\infty$. As an observation, eq. (6a) includes the height H of the planar wall and contrarily, eq. (6b) excludes H.

The exact, analytic solution of eq. (3) incorporating eqs. (6a) and (6b) is developed in the Appendix.

For the case of cooling a vertical planar wall, inspection of eqs. (6a) and (6b) reveals that the largest mean convective coefficient $\bar{h}_{max}$ is driven by the initial temperature difference $T_{in} - T_\infty$ at the initial time $t = 0$. Correspondingly, from eqs. (6a) and (6b) the two derived relations for $\bar{h}_{max}$ are

$$\bar{h}_{max} = \frac{0.59}{H^{1/4}}\left[k_f\left(\frac{g\beta}{\alpha\nu}\right)^{1/4}\right](T_{in} - T_\infty)^{1/4} \quad \text{for laminar regime} \quad (7a)$$

$$\bar{h}_{max} = 0.10\left[k_f\left(\frac{g\beta}{\alpha\nu}\right)^{1/3}\right](T_{in} - T_\infty)^{1/3} \quad \text{for turbulent regime} \quad (7b)$$

which are valid at the initial time $t = 0$.

Additionally, the lumped criterion turns into

$$Bi_l = \frac{\bar{h}_{max} L}{k_s} < 0.1.$$

and employing eqs. (7a) and (7b) hands over the inequalities

$$\frac{\bar{h}_{max}L}{k_s} = \left\{\frac{0.59}{H^{1/4}}\left[k_f\left(\frac{g\beta}{\alpha\nu}\right)^{1/4}\right](T_{in} - T_\infty)^{1/4}\right\}\frac{L}{k_s} < 0.1 \quad \text{for laminar regime} \quad (8a)$$

$$\frac{\bar{h}_{max}L}{k_s} = \left\{0.10\left[k_f\left(\frac{g\beta}{\alpha\nu}\right)^{1/3}\right](T_{in} - T_\infty)^{1/3}\right\}\frac{L}{k_s} < 0.1 \quad \text{for turbulent regime} \quad (8b)$$

To involve the thermal conductivity of the solid k_s directly, it may be convenient to disengage the solid–to–fluid thermal conductivity ratio k_s/k_f in the two preceding inequalities (8a) and (8b). This operation provides the modified inequalities

$$\frac{0.59L}{H^{1/4}}\left[\left(\frac{g\beta}{\alpha\nu}\right)^{1/4}(T_{in} - T_\infty)^{1/4}\right]\frac{k_f}{k_s} < 0.1 \quad \text{for laminar regime} \quad (9a)$$

$$0.10L\left[\left(\frac{g\beta}{\alpha\nu}\right)^{1/3}(T_{in} - T_\infty)^{1/3}\right]\frac{k_f}{k_s} < 0.1 \quad \text{for turbulent regime} \quad (9b)$$

Reorganizing terms and recognizing that $Ra_{H,in} = \frac{g\beta}{\alpha\nu}(T_{in} - T_\infty)H^3$ denotes the initial Rayleigh number in terms of the initial–to–fluid temperature difference $T_{in} - T_\infty$, i.e., the largest temperature difference during the cooling process, eqs. (9a) and (9b) can be compactly rewritten as

$$\frac{k_s}{k_f} > 5.9\frac{L}{H}Ra_{H,in}^{1/4} \quad \text{for laminar regime} \quad (10a)$$

$$\frac{k_s}{k_f} > \frac{L}{H}Ra_{H,in}^{1/3} \quad \text{for turbulent regime} \quad (10b)$$

Notice here that the solid-to-fluid thermal conductivity ratio k_s/k_f ratio varies directly proportional with a power of $Ra_{H,in}$ and inversely proportional with the slenderness ratio H/L

5. Case of Heating

For the case of heating a vertical planar wall, the largest mean convective coefficient takes place at a future target temperature T_{fut} linked to a future target time t_{fut}. Consequently, from eqs. (6a) and (6b), the proper equations for $\bar{h}_{max}$ are given by

$$\bar{h}_{max} = \frac{0.59}{H^{1/4}}\left[k_f\left(\frac{g\beta}{\alpha\nu}\right)^{1/4}\right](T_{fut} - T_\infty)^{1/4} \quad \text{for laminar regime} \quad (11a)$$

$$\bar{h}_{max} = 0.10\left[k_f\left(\frac{g\beta}{\alpha\nu}\right)^{1/3}\right](T_{fut} - T_\infty)^{1/3} \quad \text{for turbulent regime} \quad (11b)$$

and similarly from eqs. (10a) and (10b)

$$\frac{k_s}{k_f} > 5.9\frac{L}{H}Ra_{H,fut}^{1/4} \quad \text{for laminar regime} \quad (12a)$$

$$\frac{k_s}{k_f} > \frac{L}{H}Ra_{H,fut}^{1/3} \quad \text{for turbulent regime} \quad (12b)$$

6. Practical Engineering Applications

In engineering practice worldwide, the three most used coolants are air, water and oil. Correspondingly, it makes sense to scrutinize these three fluids under laminar conditions characterized by $Ra_{D,in} < 10^9$ and also turbulent conditions otherwise for $Ra_{D,in} > 10^9$.

Consider a vertical planar wall of height H and thickness $2L$ at a high initial temperature T_{in} immersed in quiescent fluid at a low free-stream temperature T_∞. The question is to explore the magnitude of the thermal conductivity of the solid k_s that makes the lumped heat equation (3) permissible when natural convection is the heat transfer mode. As a test case, let us take a vertical planar wall with a reasonable slenderness ratio H/L = 10, which is appropriate to one-dimensional analysis.

a) *Air*

The thermal conductivity of air stays around k_f = 0.026 W/m.°C at ambient temperature of 20 °C (Holman [4]). Substituting the values for H/L and k_f in eq. (10a) , implies that the thermal conductivity of the solid k_s must satisfy the inequality

$$k_s > 0.015\ Ra_{H,in}^{1/4} \quad \text{for laminar regime} \quad (13a)$$

First, focusing on the lower limit $Ra_{H,in} = 10^4$ attached to eq. (4a), results in $k_s >$ 0.15 W/m.°C. Second, at the critical laminar–turbulent threshold of $Ra_{H,in,cr} \approx 10^9$, it turns out that $k_s > 2.667$ W/m.°C. Consulting the Table of Properties for Solids in Reference [4], it is found that all metals fulfill the lumped criterion for all laminar Ra_H inside $10^4 < Ra_H < 10^9$.

Similarly, substituting the values of *H/L* and k_f in eq. (10b), the thermal conductivity of the solid k_s must satisfy the inequality

$$k_s > 0.0026\, Ra_{H,in}^{1/3} \qquad \text{for turbulent regime} \qquad (13b)$$

Turning the attention to the highest turbulent $Ra_{D,in}$ = 10^{13} in eq. (4b), yields k_s > 56.0. W/m °C. A look up at the Table of Properties for Solids in Ref. [4], reveals that most metals fulfill the lumped criterion. The metals are: Aluminum, Copper, Gold, Iron, Nickel, Silver, Tungsten and Zinc. Besides, it is observable that all Carbon steels are excluded and do not comply with the lumped criterion for air. As $Ra_{D,in}$ decreases, k_s downturns, for instance at $Ra_{D,in}$ = 10^{11}, k_s > 12.07 W/m.°C. Examining the Table of Properties for Solids in Reference [4], it is found that all metals fulfill the lumped criterion.

b) *Water*

The thermal conductivity of water stays around k_f = 0.647 W/m °C at ambient temperature of 20 °C. Substituting the values of *H/L* and k_f in eq. (10a) , indicates that the thermal conductivity of the solid k_s responds to the inequality

$$k_s > 0.382\, Ra_{H,in}^{1/4} \qquad \text{for laminar regime} \qquad (14a)$$

First, turning the attention to the lower limit for laminar natural convection $Ra_{D,in}$ = 10^4, it turns out that k_s > 3.82 W/m.°C. Second, moving next to the critical laminar–turbulent threshold at $Ra_{H,in,cr} \approx 10^9$, indicates that the solid thermal conductivity has to be k_s > 67.93 W/m.°C. A look up at the Table of Properties for Solids in Ref. [4], reveals that most metals fulfill the alternate lumped criterion. The metals are: Aluminum, Copper, Gold, Iron, Silver, Tungsten and Zinc. Further, it is observable that all Carbon steels are excluded and do not comply with the lumped criterion for water.

$$k_s > 0.065\;\; Ra_{H,in}^{1/3} \qquad \text{for turbulent regime} \qquad (14b)$$

Second, turning the attention to the upper turbulent limit of $Ra_{D,in,cr} \approx 10^{13}$, evaluating eq. (10b) gives the solid thermal conductivity k_s > 1,400.4 W/m °C. Reviewing the Table of Properties for Solids in Ref. [4], it signals that no metal meet the stringiest restriction. Incidentally, the only solid that does it is diamond. As $Ra_{D,in}$ decreases, k_s downturns, for instance at $Ra_{D,in}$ = 10^{11}, k_s > 301.7 W/m.°C. Referring to the Table of Properties for Solids in Reference [4], it is suggested that very few metals conform to the lumped criterion. In fact, there are three metals: Copper, Gold and Silver.

c) *Engine oil*

The thermal conductivity of engine oil stays around k_f = 0.145 W/m °C at ambient temperature of 20 °C. Substituting the values of *H/L* and k_f in eq. (10a) , illustrates that the thermal conductivity of the solid k_s relates to the inequality

$$k_s > 0.086\, Ra_{H,in}^{1/4} \qquad \text{for laminar regime} \qquad (15a)$$

First, turning the attention to the lower limit for laminar natural convection $Ra_{D,in}$ = 10^4, proves that k_s > 0.86 W/m.°C. Second, moving next to the critical laminar–turbulent threshold at $Ra_{H,in,cr} \approx 10^9$, demonstrates that the solid thermal conductivity k_s > 15.29 W/m.°C. Regarding the Table of Properties for Solids in Ref. [4], it is clear that most metals fulfill the lumped model criterion for laminar natural convection in water.

$$k_s > 0.015\;\; Ra_{H,in}^{1/3} \qquad \text{for turbulent regime} \qquad (15b)$$

Second, turning the attention to the upper turbulent limit of $Ra_{D,in,cr} \approx 10^{13}$, evaluating eq. (10b) conveys the solid thermal conductivity k_s > 323.2 W/m °C. Inspecting the Table of Properties for Solids in Reference [4], it is divulged that few solids fulfill the lumped criterion. They are Copper, Gold and Silver. As $Ra_{D,in}$ decreases, k_s downturns, for instance at $Ra_{D,in}$ = 10^{11}, k_s > 69.6 W/m.°C. A look up at the Table of Properties for Solids in Ref. [4], reports that most metals satisfy the lumped criterion. The metals are: Aluminum, Copper, Gold, Iron, Silver, Tungsten and Zinc. Further, it is observable that all Carbon steels are excluded and do not adhere to the lumped criterion.

7. Conclusions and Recommendations

In order to use the lumped heat equation (2) implicating that the lumped Biot number $Bi_l = \frac{\bar{h}}{k_s}\left(\frac{V}{A}\right) < 0.1$, the traditional procedure comprises three steps: 1) evaluate $\bar{h}$ from the proper correlation equations, 2) read the value of the thermal conductivity k_s of the solid in a Table of Properties for Solids and 3) calculate Bi_l and compare against 0.1. For the situation involving a vertical planar wall with natural convection cooling, an equivalent lumped criterion stated in eqs. (10a) and (10b) gives rise to a simpler relation between the solid-to-fluid thermal conductivity ratio k_s/k_f and the initial Rayleigh number, $Ra_{D,in}$. Within this alternative framework, a list of candidate solids can be easily identified for laminar and turbulent natural convection in fluid environments, such air, water and oil. The opposite situation when a vertical planar wall is heated by natural convection can be treated in a similar way, but the analysis is turns out to be a bit more involved.

Nomenclature

A	surface area, m^2
Bi_l	lumped Biot number, $\frac{\bar{h}}{k_s}\left(\frac{V}{A}\right)$
Bi_l	lumped Biot number for vertical planar wall, $\frac{\bar{h}L}{k_s}$
c_v	specific heat capacity at constant volume, J/kg ºC
c_p	specific heat capacity at constant pressure, J/kg ºC
C	constant in eq. (A.2)
g	gravitational acceleration, m/s^2
$\bar{h}$	mean convection coefficient, W/m^2 ºC
H	height of planar wall, m
k	thermal conductivity, W/m ºC
L	semi-thickness of planar wall, m

n exponent in eq. (A.2)

$\overline{Nu}_H$ mean Nusselt number, $\frac{\bar{h}H}{k_f}$

Pr Prandtl number, $\frac{\mu c_p}{k_f}$

Ra_H Rayleigh number, $\frac{g\beta}{\alpha\nu}(T - T_\infty)H^3$

$Ra_{H,in}$ initial Rayleigh number, $\frac{g\beta}{\alpha\nu}(T_{in} - T_\infty)H^3$

t time, s

$\bar{T}$ mean temperature, ºC

T_{in} initial temperature, ºC

T_f film temperature, ºC

T_{fut} future temperature, ºC

T_∞ fluid temperature, ºC

V volume, m^3

Greek Symbols

α thermal diffusivity, m^2/s

β coefficient of volumetric thermal expansion, K^{-1}

$\bar{\theta}$ mean temperature excess, $\bar{T} - T_\infty$, ºC

μ dynamic viscosity, N s/m^2

ν kinematic viscosity, m^2/s

Subscripts

fut refers to future

in refers to initial

s refers to solid

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Appendix: Lumped heat equation

From eq. (3), the lumped heat equation for a vertical planar wall sketched in Fig. A.1 along with the initial condition are

$$\rho c_v L \frac{d\bar{T}}{dt} = -\bar{h}(\bar{T} - T_\infty), \quad \bar{T} = T_{in}, t = 0 \qquad \text{(A.1)}$$

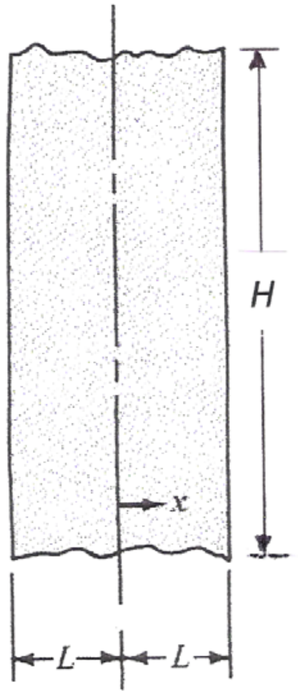

Fig. A.1. Vertical planar wall configuration.

From the correlation equations (4a) and (4b), the mean convection coefficient $\bar{h}$ for a vertical planar wall can be expressed in compact form as

$$\bar{h} = C(\bar{T} - T_\infty)^n \qquad \text{(A.2)}$$

where in general the exponent n = ¼ identifies laminar regime and n = 1/3 turbulent regime.

Introducing the mean temperature excess

$$\bar{\theta} = \bar{T} - T_\infty$$

the lumped heat equation (A.1) and the initial condition transform into

$$\rho c_v L \frac{d\bar{\theta}}{dt} = -C\bar{\theta}^{n+1}, \quad \bar{\theta} = \theta_{in}, t = 0 \qquad \text{(A.3)}$$

Separating variables and introducing the initial condition gives

$$\frac{1}{n}\left(\frac{1}{\bar{\theta}^n} - \frac{1}{\theta_{in}{}^n}\right) = \left(\frac{C}{\rho c_v L}\right) t$$
(A.4)

or equivalently

$$\left[\frac{1}{(\bar{T} - T_\infty)^n} - \frac{1}{(T_{in} - T_\infty)^n}\right] = \left(\frac{nC}{\rho c_v L}\right) t$$
(A.5)

After rearranging terms, the mean temperature distribution $\bar{T}(t)$ turns out to be

$$\bar{T}(t) = T_\infty + \left[\frac{1}{\left(\frac{nC}{\rho c_v L}\right) t + (T_{in} - T_\infty)^{-n}}\right]^{1/n} \quad \text{(A.6)}$$

Subsequently, eq. (A.6) can be particularized first for laminar regime characterized with *n* = ¼, resulting in

$$\bar{T}(t) = T_\infty + \left[\frac{1}{\left(\frac{C}{4\rho c_v L}\right) t + (T_{in} - T_\infty)^{-1/4}}\right]^{4} \quad \text{(A.7a)}$$

and second for turbulent regime characterized with *n* = 1/3, resulting in

$$\bar{T}(t) = T_\infty + \left[\frac{1}{\left(\frac{C}{3\rho c_v L}\right) t + (T_{in} - T_\infty)^{-1/3}}\right]^{3} \quad \text{(A.7b)}$$

As a point of reference, for the case of forced convection cooling a vertical planar wall embodying constant $\bar{h}$, the mean temperature distribution $\bar{T}(t)$ is

$$\bar{T}(t) = T_\infty + (T_{in} - T_\infty) \exp\left(-\frac{\bar{h}}{\rho c_v L} t\right) \quad \text{(A.8)}$$

Upon comparing eqs. (A.6) and (A.8), it is clear that the structure of the former for natural convection is more complex than the structure of the latter for forced convection.