Int. J. of Thermal & Environmental Engineering
Volume 10, No. 2(2015) 87-93

Theory and Design of Counter Flow Shell-and-Coil Heat Exchanger for CO_2 Based Solar Water Heater

N. Abas[a,*], N. Khan[a,b]

[a] *Department of Electrical Engineering, COMSATS Institute of Information Technology, Islamabad, Pakistan*
[b] *Department of Electrical Engineering, University of Gujrat, Hafiz Hayat Campus, Gujrat, Pakistan*

Abstract

A shell type helical tube countercurrent flow heat exchanger was designed, fabricated and tested using CO_2 refrigerant material. Helical copper tube was immersed in bulk water tank. Hot CO_2 was piped to upper coil baffle and cold CO_2 was circulated into collector loop through lower baffle. Cold water was made to enter through lower tap and hot water was taken out from upper tap. Both in/out pipes were inserted from the top and coil tubes from upper and lower sides of water tank. Supercritical thermosyphon operation was achieved by evacuated glass tube solar heat collector using CO_2 refrigerant. U shaped copper pipes enveloped in aluminum foil were inserted in evacuated glass tubes to transfer heat under thermosiphon principle to upper header connected to inlet baffle of shell type helical coil heat exchanger. Lower header was connected to heat exchanger helical coil outlet baffle. Solar collector heat raised CO_2 refrigerant temperature from 35 to 78°C giving temperature difference of 43°C. Temperature of CO_2 refrigerant at exit from heat exchanger was found to be 40°C at surrounding ambient temperature of 36°C. Heat exchanger raised the inlet water temperature from 26 to 55°C under off water tap condition in about 3 hours. Inlet and outlet temperature difference of heat exchanger was measured to be 27°C. We believe system efficiency can further increase if we use the system in mild sunshine cold weather regions duplicating geothermal loop.

Keywords: *Heat exchanger, Shell & Helical Tube, Counter flow, Countercurrent.*

1. Introduction

Heat exchangers efficiently transfer thermal energy from heating (hot) to heated (cold) fluids. Heating and heated fluids carrying metal tubes are thermally connected to avoid physical mixing of fluids. Natural countercurrent flow heat exchange biological processes include human breathing (nasal passages), animal carotid rete in hoofed animals and blood circulation in wading birds and fishes. Heat exchange industrial processes include space heating, engine oil-fuel flow, refrigeration and air conditioning. Any heat exchanger may take counter flow (countercurrent) or parallel flow (concurrent) horizontal or vertical configurations. High pressure heat exchangers such as supercritical CO_2 working over 70-bar usually take the form of a shell and tube coil. Coil folds, baffle, tube pitch and diameter size depend on heating demand. Thin tube design remains efficient and economic but fouls up fast.

Thermosiphon principle based systems are gravity driven but addition of circulation pump increases efficiency. Space constraint designs usually go for self cleaning Spiral Heat Exchangers (SHE) in pursuance of technical and economic tradeoffs. Heating and heated fluids flow in opposite directions in countercurrent flow heat exchangers.

Shell-and-tube coil heat exchangers are widely used in petrochemical and power generation industries[1]. Lot of energy can be saved by improving the thermal performance of heat exchangers. Usually baffle shell and tube heat exchangers have

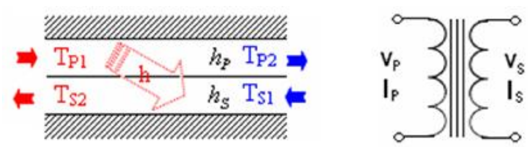
* Corresponding author. Tel.: +923005879115
E-mail: naeemk56@yahoo.com

DOI: 10.5383/ijtee.10.02.001

high pressure loss[2] and large dead flow regions. Orifice plates reduce dead flow regions as well as fouling. Counter flow is generally preferred over parallel flow heat exchangers[3]. Several researchers have reported theoretical and experimental studies of heat transfer characteristics of helical heat exchangers for solar water heater applications[4,5]. Shell-and-coil heat exchangers are used in domestic solar water heaters[6,7.] Helical coil tubes have high heat transfer coefficients compared to straight tubes[8]. Natural convection driven solar water heaters employing supercritical CO_2 as working fluids are under extensive research[9] using helical coil counter flow heat exchangers and U-tube[10] collectors. Supercritical CO_2 cycle may be used for low and high temperature heat sources[11] for thermal battery smart grid applications [12].

2. A Counter Flow Heat Exchanger

Bearing in mind conventional electrical transformer model all quantities related to heating fluids (hot) will be represented by subscript p for primary and those of heated fluids (cold) will be designated by subscript s for secondary. Entrance and exit side quantities will be designated by numbers 1 and 2 as illustrated in Fig.1.



a. Counter flow heat exchanger b. Transformer symbol
Fig.1 Counter flow heat exchanger analogy with a transformer

This model is based on overall heat exchanger design procedure following Donatello's approach[13]. Heat energy (Q) transferred by hot fluid at temperature T_P through surface area S with heat transfer coefficient h to cold fluid at temperature T_S may be given by

$$q = US(T_P - T_S) = uS\Delta T \tag{1}$$

Overall heat transfer coefficient at outer surface of heat exchanger may be given by

$$U_O = 1/(1/h_P + x_w d_O / kd_m + d_O / h_S d_i) \tag{2}$$

Where h_P and h_S are heat transfer coefficients (W/m^{2}K) of heating and heated fluids; x_w is tube wall's thickness; k (W/mK) is tube's thermal conductivity; d_i, d_o and d_m are internal, outer and medium diameters of the tubes. Similarly, heat transfer coefficient at inner surface of heat exchanger is

$$U_i = 1/(1/h_P + x_w d_i / kd_m + d_i / h_S d_i) \tag{3}$$

If temperature difference ΔT is replaced by mean temperature difference ΔT_m then (1) may be rewritten as

$$q = US\Delta T_m \tag{4}$$

The mean isobaric specific heat (c_{pm}) is given by

$$c_{pm} = \int_{T_S}^{T_P} c_p dt / (T_P - T_S) \tag{5}$$

Temperatures T_P and T_S correspond to enthalpies E_P and E_S respectively. Therefore,

$$c_{pm} = [(E_P - E_S)/(h_P - h_S)] \times 1000 \tag{6}$$

Enthalpy of a fluid may be expressed by general equation

$$h = XT + YT^2 + ZT^3 \tag{7}$$

Enthalpy equations for various natural refrigerants such as water (steam), air, nitrogen and CO_2 are given bellow:

For water at 20 to 250°C

$$h = 421.96\frac{T}{100} - 9.36\left(\frac{T}{100}\right)^2 + 5.74\left(\frac{T}{100}\right)^3 \tag{8}$$

For air at 0 to 300°C

$$h = 1003.79\frac{T}{1000} + 37.76\left(\frac{T}{1000}\right)^2 + 72\left(\frac{T}{100}\right)^3 \tag{9}$$

For nitrogen at 0 to 500°C

$$h = 1038\frac{T}{1000} + 18.4\left(\frac{T}{1000}\right)^2 + 78.13\left(\frac{T}{100}\right)^3 \tag{10}$$

For CO_2 at 0 to 500°C

$$h = 813.3\frac{T}{1000} + 502.3\left(\frac{T}{1000}\right)^2 - 209.5\left(\frac{T}{100}\right)^3 \tag{11}$$

Fluid parameters like enthalpy (h), density (ρ) and heat capacity (c_p) change with change in temperatures and are usually calculated by empirical equations obtained by experimental measurements.

Thermal balance equation for heating and heated fluids energy in counter flow helical coil heat exchanger may be expressed as,

$$M_S c_{pm}^S (T_{S2} - T_{S1}) = \eta_e M_P c_{pm}^P (T_{P1} - T_{P2}) \tag{12}$$

Where M_S and M_P are mass flow rates of heated and heating fluids, η_e is heat transfer efficiency, c_{pm}^S and c_{pm}^P are mean isobaric specific heats of heated and heating fluids. Considering an elementary surface area dS the heat transferred across may be given by

$$dq = UdS(T_P - T_S) \tag{13}$$

Heat change (dq) in heating fluid

$$dq = -\eta_e M_P c_{pm}^P dT_P \tag{14}$$

Heat change (dq) in heated fluid

$$dq = -M_S c_{pm}^S dT_S \tag{15}$$

Net heat transfer from heating to head fluid may be given by

$$d(T_P - T_S) = -UdS(T_P - T_S)\left(\frac{1}{\eta_e M_P c_{pm}^P} - \frac{1}{M_S c_{pm}^S}\right) \tag{16}$$

From (12) and (15) we can write

$$\frac{1}{\eta_e M_P c_{pm}^P} - \frac{1}{M_S c_{pm}^S} = \frac{1}{q}(T_{P1} - T_{S2} - T_{P2} + T_{S1}) \tag{17}$$

Letting $\Delta T_I = T_{P1} - T_{S2}$ (greatest temperature difference GTD) and $\Delta T_{II} = T_{P2} - T_{S1}$ (lowest temperature difference LTD) above equation becomes

$$\frac{1}{\eta_e M_P c_{pm}^P} - \frac{1}{M_S c_{pm}^S} = \frac{\Delta T_I - \Delta T_{II}}{q} = \frac{GTD - LTD}{q} \tag{18}$$

From (15), (16) and (17) we obtain

$$\frac{d(T_P - T_S)}{T_P - T_S} = -\frac{UdS}{q}(\Delta T_I - \Delta T_{II}) \tag{19}$$

Integration of (19) gives

$$\left| -\log_e (T_P - T_S) \right|_I^{II} = -\frac{US}{q}(\Delta T_I - \Delta T_{II}) \tag{20}$$

Or

$$\log_e \frac{\Delta T_I}{\Delta T_{II}} = \frac{US}{q}(\Delta T_I - \Delta T_{II}) \tag{21}$$

Solution of (21) gives

$$q = US(\Delta T_I - \Delta T_{II}) / \log_e \frac{\Delta T_I}{\Delta T_{II}} = uS\Delta T_{ml} \tag{22}$$

Where the mean logarithmic temperature difference ΔT_{ml} (MLTD) is given by

$$\Delta T_{ml} = (\Delta T_I - \Delta T_{II}) / \log_e \frac{\Delta T_I}{\Delta T_{II}} \tag{23}$$

Typical values of mean logarithmic temperature difference ΔT_{ml} in thermosyphon based heat exchangers is not very therefore the ΔT_{ml} does not differ very much even if the arithmetic mean $\Delta T$is used instead ΔT_{ml}. In case of ΔT it may be estimated by

$$\Delta T = (\Delta T_I + \Delta T_{II})/2 \tag{24}$$

ΔT_{ml} may be related to ΔT by

$$\Delta T_{ml} = \chi (\Delta T_I + \Delta T_{II})/2 = \chi . \Delta T_m \tag{25}$$

The correcting factor χ may be given by

$$\chi = 2(\Delta T_I - \Delta T_{II})/(\Delta T_I + \Delta T_{II}) \log_e \frac{\Delta T_I}{\Delta T_{II}} \tag{26}$$

The correcting factor "χ" is unity when $\Delta T_I/\Delta T_{II}$ is 1 but it reduces down to 0.71 when $\Delta T_I/\Delta T_{II}$ approaches 10 for large temperature differences.

Log mean temperature difference (LMTD) obtained by (25) is good for counter flow heat exchangers. In case of shell-and-coil type counter flow heat exchangers the LMTD needs further correction factor χ_c.
We know mean logarithmic temperature for heating fluid is given by

$$T_{ml}^P = (T_{P1} - T_{P2}) / \log_e (T_{S1} / T_{S2}) \tag{27}$$

To deal with real situations, for calculating mean temperature ΔT_m, let us define three parameters as follows

$$\alpha = (T_{P2} - T_{S1})/(T_{S1} - T_{S2}) \tag{28}$$

$$= \eta_e M_P c_{pm}^P / M_S c_{pm}^S \tag{29}$$

$$\gamma = US / \eta_e M_P c_{pm}^P \tag{30}$$

As temperature gauges display inlet and outlet temperatures therefore parameter α can be computed to be known. Parameter β can also be calculated from available data therefore it may be regarded as known. Parameter γ for the counter flow heat exchanger for $\beta \neq 1$may be given by[13].

$$\gamma_C = \frac{1}{1-} \log_e \left(\frac{1-}{\alpha} + \right) \tag{31}$$

In case of $\beta = 1$ γ_c in terms of parameter α may be given by

$$\gamma_C = \frac{1}{\alpha} - 1 \tag{32}$$

We know transferred heat in heat exchanger may be given by

$$q = \eta_e M_P c_{pm}^P (T_{P1} - T_{P2}) = \eta_e M_P c_{pm}^P \gamma \Delta T_m = US\Delta T_m \tag{33}$$

Compare terms within (33) ΔT_m becomes

$$\Delta T_m = (T_{P1} - T_{P2})/\gamma \tag{34}$$

In case of counter flow heat exchangers

$$\chi_C = \frac{\Delta T_m}{\Delta T_{ml}} = \frac{\gamma_C}{\gamma} \tag{35}$$

Values of correction factor parameter χ_c for different known values of α and β for ten sections helical coil heat exchanger are shown in Table 1[13].

Table 1. Correction factors (¡ c) for ten sections counter flow helical coil HE (Data from reference [13])

↓β α→	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48	0.52	0.56	0.60
0.4	0.985														
0.5	0.977	0.987													
0.6		0.980	0.988												
0.7		0.969	0.982	0.988											
0.8			0.971	0.982	0.988										
0.9			0.953	0.972	0.982	0.988									
1.0				0.952	0.972	0.983	0.988								
1.1					0.950	0.972	0.983	0.989							
1.2						0.948	0.973	0.084							
1.3							0.947	0.974	0.985						
1.4								0.947	0.977	0.987					
1.5									0.950	0.980	0.989				
1.6										0.960	0.984				
1.7											0.971	0.988			
1.8												0.980			
1.9												0.947	0.987		
2.0													0.977		
2.2														0.979	
2.4															0.985

3. Experimental Setup

Solar water heater as shown in Fi.g. 2, using CO_2 as working fluid exhibited following temperatures after three hours exposure to sunlight on October 1, 2012.

Fig. 2 Experimental set

Table 2. Heat exchanger helical copper tube parameters

Tube size (m)	=	0.009525
Tube wall thickness (m)	=	0.00075
Tube size (m)	=	0.008025
Tube length (m)	=	5.334
Tube outer surface area (m^2)	=	0.15953

Copper tubing surface area was calculated using $A = \pi \times D \times L$

Table 3. Solar collector aperture area of glass tubes

Evacuated glass tube length (m)	=	1.8
Tube outer diameter (m)	=	0.00058
Tube inner diameter [m]	=	0.00047
Evacuated glass surface area (m^2)	=	0.0016390
Total tubes	=	09
Tubes collector surface area (m^2)	=	1.475172

Solar collector aperture area was calculated using $A = \pi \times r \times L$

3.1. Temperature gauges readings

CO_2 temperature (T_{P1}) at entrance of heat exchanger = 78°C

CO_2 temperature (T_{P2}) at exit of heat exchanger (HE) = 40°C

Water tank initial temperature (T_{S1}) at entrance of HE = 26°C

Water tank initial temperature (T_{S2}) at exit of HE = 55°C

3.2. Calculation of GTD (ΔT_L) and LTD (ΔT_o) values

GTD = $\Delta T_L = (T_{P2} - T_{S1}) = (40 - 26) = 14°C$

LTD = $\Delta T_o = (T_{P1} - T_{S2}) = (78 - 55) = 23°C$

$$LMTD = \frac{GTD - LTD}{\ln(GTD/LTD)} = \frac{14-23}{\ln(14/23)} = \frac{-9}{-0.49659} = 18.12$$

What if GTD and LTD values are even reversed by error it makes no difference on computation of LMTD.

GTD = $T_{P1} - T_{S2}$ = 78-55 = 23

LTD = $T_{P2} - T_{S1}$ = 40 – 26 = 14

$$LMTD = \frac{GTD - LTD}{\ln(GTD/LTD)} = \frac{23-14}{\ln(23/14)} = \frac{9}{0.4964} = 18.13$$

Practically GTD becomes LTD if measurements are taken on idle heat exchanger.

4. Conclusion

A gravity driven thermosiphon solar water heating system is developed to harness solar insolation in low sunshine regions. This innovatory system uses CO_2 as working fluid to collect mild sunlight to heat the water in ice cold areas. Carbon dioxide refrigerant exhibits supercritical heat transfer properties at 7.38MPa pressure and 31.1°C temperature. This solar water heater harnesses solar energy by fitting U shaped copper heat removal pipes in evacuated glass tubes.

Each U shaped copper tube is connected to upper and lower headers between solar collector and heat exchanger. This system works automatically by natural thermosiphon circulation force caused by density difference of CO_2 at different temperatures. Carbon dioxide refrigerant easily attains 75°C during 30 to 35°C ambient temperatures. When the hot refrigerant is passed through shell-and-coil type counter flow heat exchanger the inlet water temperature increases from 26 to 55° giving off temperature gradient of 29°C.

Maximum temperature difference in heat exchanger is 52°C. Solar insulation acts as driving force starting thermosiphon effect in CO_2. This system provides 23°C greatest temperature difference (GTD), 14°C lowest temperature difference (LTD) and 18.13°C log mean temperature difference (LMTD).

Special arrangement in manifolds and inside the evacuated tubes makes it possible to stop reverse thermosyphon. This innovatory solar water heater can perform in subzero temperature areas where water based systems cease to function after freezing.

Nomenclature

c	specific heat (J/kgK)
α	dimensionless parameter
d	diameter (m)
β	dimensionless parameter
E	specific enthalpy (kJ/kg)
γ	dimensionless parameter
g	acceleration of gravity (m/s^2)
χ	correction factor
h	heat transfer coefficient (W/m^{2}K)
η	efficiency (%)
k	thermal conductivity (W/mK)
η_e	heat exchange efficiency
L	helical coil tube length (m)
κ	ratio of ψ and M
M	mass flow rate (kg/s)
ρ	density (kg/m^3)
Nu	Nusselt number
Ψ	thermal constant per unit length (m^{-1})
P	pressure (Pa)
θ	collector inclination angle
Pr	Prandtl number
δ	Curvature ratio (r/R)

References

[1] Yonghua Y, Aiwu F, Xuejiang L, Suyi H, Wei L. Experimental and numerical investigations of shell-side thermo-hydraulic performances for shell-and-tube heat exchanger with trefoil-hole baffles. Appl. Therm. Engg. 2013;50:950-956.

[2] Mandal MM, Nigan KDP. Experimental study on pressure drop and heat transfer of turbulent flow in tube in helical heat exchanger. Industrial and Engineering Chemistry 2009;48:9318-9324.

[3] Taherian H, Allen PL. Experimental study of natural convection shell and coil type heat exchangers. ASME, HTD 1998;357:31-38.

[4] Jung-Jung, 2012[

[5] Koffi PME, Andoh HY, Gbaha HY, Toure S, Ado G. Theoretical and experimental study of solar water heater with internal heat exchanger using thermosiphon system. Energy Conversion & Management 2008;49:2279-2290.

[6] Nasser G, Hessam T, Mofid G, Hessam M. An experimental study of thermal performance of shell-and-coil heat exchanger. *International Communications in Heat and Mass Transfer* 2010;**37**:375-781.

[7] Close DJ. The performance of solar water heaters with natural circulations. Solar Energy 1962;6:30-40.

[8] Rahul K, Nitim B, Jha RS. Development of heat transfer coefficient correlation for concentric helical coil heat exchangers. Int. J. Thermal Science 2009;48:2300-2308.

[9] Wonseok K, Jongmin C , Honghyun C. Performance analysis of hybrid solar geothermal CO2 heat pump

system for residential heating. Renewable Energy 2013;50:596-604.

[10] Georgios AF, Paul C, Panayiotis P. Single and double U-tube groung heat exchangers in multiple layers substrates. Applied Energy 2012;Online since.

[11] Kim YM, Kim CG, Favrat D. Transcritical or supercritical CO2 cycles using low and high temperature heat hources. Energy 2012;43:402-415.

[12] Morten BB, Kazuaki Y, Ali S, Carolina C. Thermal battery with CO2 compression heat pump: Techno-economic optimization of a high efficiency smart grid option for buildings. Energy and Buildings 2012;50:128-138.

[13] Annaratone D, Handbook for Heat Exchangers and Tube Banks Design. Springer, 2010.