

## Design of a Bubble Pump Cooling System Demonstration Unit

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### Abstract

The aim of this study is to examine the feasibility of developing a bubble pump cooling system prototype that can dissipate a certain amount of heat generated by an insulated gate bipolar transistor. To this end, a bubble pump cooling system prototype was designed and developed. The performance of the bubble heat pump cooling system prototype is described by presenting some experimental test data. This paper provides details about the design and development of the heat pump cooling system prototype, as well as testing and validation of this unit.

**Keywords:** *Bubble Pump, Design, Cooling System*

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### 1. Introduction

Several methods in which vapor bubbles or gas can be used for the pumping of liquids are described in the literature [1-8]. Bubble pumps or air-lift pumps are devices that raise a liquid using vapor bubbles or compressed air, or in general raise a liquid by introducing bubbles into the outlet tube. The hydrostatic principle is used to create the required pressure difference between the pressure in the outlet tube and the pressure at the tube opening. The hydrostatic pressure in the outlet tube is reduced versus the hydrostatic pressure at the tube opening. The bubble pump depends on the saturation temperature (i.e., boiling point) of the working fluid for its operation.

A thermally driven bubble pump that can be powered by solar thermal energy or waste heat can be used to lift the liquid solution without any mechanical moving part. Pfaff et al. [9] carried out both analytical and experimental studies on bubble pump. A test rig was built in glass to examine the performance of the bubble pump, to visualize the flow regime and to validate the analytical model. Several studies [10-15] have examined the performance of the bubble pump for diffusion absorption cooling machines and refrigeration units.

Precision Cooling Division of Parker Hannifin Corporation has requested the development of a bubble pump cooling system demonstration unit. The bubble pump cooling system will be used to show potential customers of Parker Hannifin Corporation, their ability to develop a system that can dissipate heat from an insulated gate bipolar transistor (IGBT). This bubble pump will be driven independently of any other power source and will operate from the thermal energy losses of the IGBT. A schematic of the cooling system is shown in Fig. 1.

This cooling system's cycle operates with a fluid that is heated to two-phase and is driven by the bubble pump. The bubble pump cooling system works by maintaining the fluid near the saturated state and when introduced to a heat source it begins to boil. Vapor bubbles in the boiling fluid increase in size as the quality or the amount of vapor increases and becomes more buoyant than the liquid causing them to rise. The vapor bubbles rise upward through a tube with an inside diameter equivalent to the diameter of the vapor bubbles such that liquid is trapped and lifted between vapor bubbles generating flow. The two-phase fluid is then condensed back to a saturated or slightly sub-cooled liquid by rejecting heat to the surroundings. This paper provides details about the design and development of the bubble pump cooling system prototype, as well as testing and validation of this unit.

### 2. The Design Process and Specifications

The design process that was employed is the one outlined by Bejan et al. [16] and Jaluria [17]. The first essential and basic feature of this process is the formulation of the problem statement. The formulation of the design problem statement involves determining the requirements of the system, the given parameters, the design variables, any limitations or constraints, and any additional considerations arising from safety, financial, environmental, or other concerns. The following is a summary of the guidelines:

- Heat dissipation: The current system will need to dissipate 400W of power through the use of conduction cooling.
- The bubble pump must be an independent unit that is driven only by the heat source in which it will be

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removing heat. The bubble pump must not have any moving parts

- The demonstration unit must have the capability to measure and display properties including pressures and temperatures along with the ability to calculate and display quantities including the rate of heat transfer, thermal resistance, and volume flow rate with the use of a data acquisition system.
- Evaporator – The evaporator must be a cold plate to which the IGBT or power resistor can be mounted to. The surface temperature of the cold plate must be kept below 80°C.
- The heat absorbed by the evaporator from the IGBT or heat source must be transferred, with a condenser, to the surrounding air.
- Portability and demonstrate ability of the unit: This heat exchange unit will be shown to potential customers and will need to be a size that can be transported to these customers for demonstration purposes. The current requirement is that the evaporator, bubble pump, condenser, associated plumbing materials, heating and measurement devices fit in a container that is no larger than 12" high, 8" long, and 6" wide.

After the problem statement was formulated, several conceptual designs were considered and evaluated. Each design concept was evaluated by the following criteria: Effectiveness, Cost, Safety, and Size.

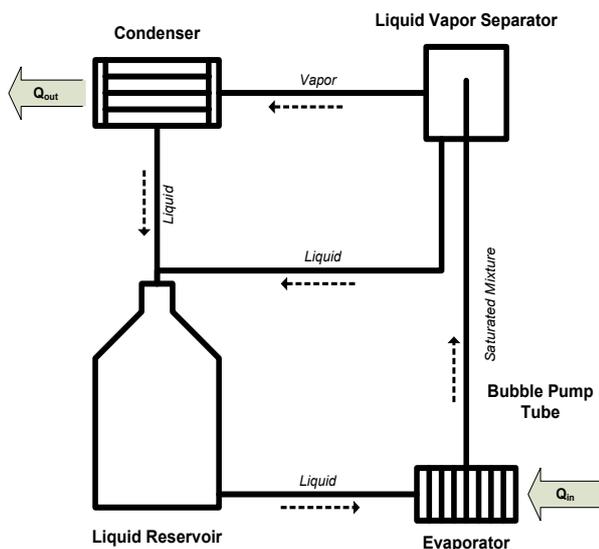


Fig.1. Schematic of the bubble pump cooling system

### 2.1. Working Fluid

An important design component to be considered for the bubble pump cooling system is the working fluid. The bubble pump cooling system transfers heat from the evaporator to the condenser by circulating the fluid in a closed loop. The system contains a specific amount of fluid so that the bubble pump is always partially filled with liquid. As the fluid absorbs heat in the evaporator it begins to boil. Vapor bubbles exiting the evaporator enter the bubble pump. The bubble pump is

essentially a vertical tube with an inside diameter relative to the size of the vapor bubbles. Because the evaporator is placed near the base of the bubble pump the more buoyant vapor bubbles rise. Liquid in the bubble pump is trapped and lift by the rising vapor bubbles to a liquid vapor separator. The liquid is then returned to a liquid reservoir and the vapor enters the condenser. The vapor is then condensed back to a liquid by transferring heat from the fluid to the surroundings. The liquid leaving the condenser returns to the liquid reservoir. The liquid reservoir replenishes fluid that is displaced in the evaporator and the bubble pump. Determining a proper working fluid for the bubble pump cooling system is essential for the system's functionality. Several fluids were considered and evaluated and the methanol was selected for this system.

### 2.2. Evaporator/Cold Plate

The IGBT is mounted to the evaporator/cold plate and uses contact cooling to transfer heat from the IGBT to the working fluid of the bubble pump cooling system. As the fluid flows through the cold plate it absorbs heat from the IGBT and will begin to vaporize. As the fluid begins to change phases from a saturated liquid to a two phase liquid vapor bubbles will begin to form. The outlet of the cold plate will be connected to the inlet of bubble pump tube. The cold plate selection will be based on its ability to remove the required heat from the IGBT and on its overall size. The cold plate used must have a low thermal resistance to meet the cooling requirements of 400W. Several concepts for the evaporator were considered and evaluated and the flat tube cold plate was selected for this system.

### 2.3. Bubble Pump

The bubble pump is a tube that is connected to the outlet of the evaporator. The vapor liquid mixture through the bubble pump tube must have a specific flow regimen for the flow to work properly. There are five basic flows that occur inside the bubble pump tube which are bubbly flow, slug flow, annular flow, transition flow, and mist flow. The optimal flow for this type of system is the slug flow regimen. The slug flow regimen consists of air bubble with a diameter that is about equivalent to the interior of the bubble pump. This air bubble forces the liquid above it to a higher point in the tube. This cycle is repeated down the tube which causes the bubble pump tube to react similar to a piston cylinder configuration. The vapor bubbles force the liquid through the tube and into the liquid vapor separator creating the flow rate of the system. It was decided that a single tube will be used for this system.

### 2.4. Liquid Vapor Separator

The liquid vapor separator is connected to the outlet of the bubble pump and is used to feed the working fluid into the reservoir or the condenser depending upon the state of the fluid. The fluid that enters into the separator that is a liquid state flows to the base of the separator where it is then directed back into the reservoir. The vapor that enters the separator will be removed by a tube in the upper part of the separator which will transfer it to the condenser. Several concepts for the liquid vapor separator were considered and evaluated and an "inline separator" was selected for this system. As shown in Fig. 2, the two-phase fluid enters from the bubble pump from the side of the separator. As the fluid enters through the side of the tube the saturated liquid will exit to the reservoir while the vapor will be directed to the condenser.

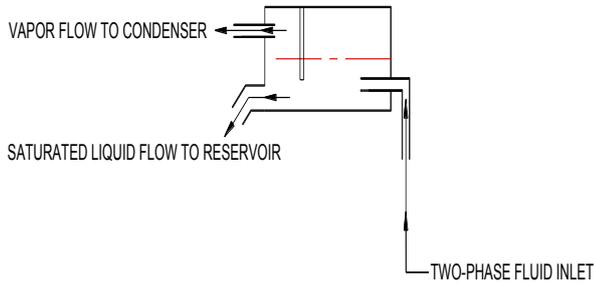


Fig.2. Schematic diagram of the inline separator

### 3. Background and Theory

The lift tube of the bubble pump cooling system is where bubbles leaving the cold plate merge together to form larger bubbles. As the bubbles rise in the lift tube, the hydrostatic pressure from the surrounding liquid decreases causing the bubbles to expand to the outer diameter of the tube forming a slug. The buoyancy force from the vapor slugs will lift small amounts of liquid up to the top of lift tube. The small amounts of liquid will be pushed out of the lift tube and into the liquid vapor separator. The liquid will be returned back to the reservoir while the vapor moves into a condenser. Once the vapor is condensed it will return back to the reservoir due to gravitational acceleration pulling the liquid downward. In order for this to occur the diameter of the bubble pump must be sized correctly for the heat load to which is applied to the cold plate. Also, the correct submergence ratio, or the ratio of the initial liquid level to the height of the lift tube, must be determined such that liquid flow can be generated.

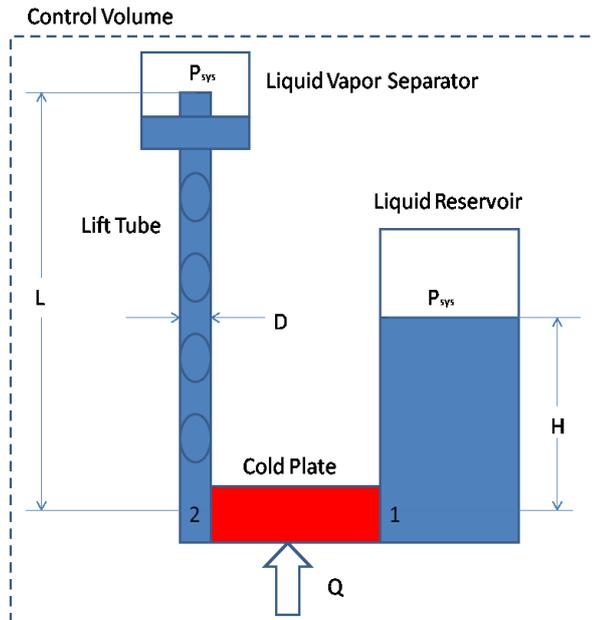


Fig.3. Bubble pump schematic

The liquid flow rate of the bubble pump was analyzed at different lift tube diameters and submergence ratios to determine the appropriate dimensions for the tube (refer to the bubble pump schematic shown in Fig. 3). For demonstration purposes the lift tube for the system will be made out of polycarbonate plastic which is transparent and does not chemically react with methanol. The initial lift tube inside diameters was selected for the study using the following equation, which predicts the maximum diameter in which slug flow occurs [18]. If the diameter of the lift tube is larger than predicted by the equation, slug flow may not occur. The lift tube diameter was then decreased until the analysis predicted no liquid flow at all submergence ratios meaning annular flow in the lift tube. In the annular flow regime a thin wall of liquid is formed around the tube wall and vapor moves through the center of the tube at high velocity.

$$D_{max} \leq 19 \sqrt{\frac{\sigma v_f}{g \left(1 - \frac{v_f}{v_g}\right)}} \approx 30 \text{ mm} \quad (1)$$

Slug flow in the lift tube was assumed for this analysis. Also, minor pressure losses throughout the system were assumed to be small and were neglected. For this analysis the minor pressure losses included losses due to geometric changes in cross-sectional area and direction of the flow path. The fluid was assumed to be at saturation temperature throughout the bubble pump cooling system. The fluid from the condenser outlet to the inlet of the cold plate was assumed to be all liquid. The saturation temperature assumption has been made because if the fluid in the lift tube is sub-cooled, vapor bubbles will condense in the lift tube rather than expand as they rise. Condensing of the vapor bubbles will occur during start up but once the system is operating at steady state, the fluid temperature should be uniformly saturated. Frictional pressure losses across the length of the cold plate were assumed to be small and also neglected.

To analytically model the system Bernoulli's equation was used to determine a relationship between the fluid pressure and velocity at the inlet of the cold plate in relation to pressure and velocity at upper portion of the liquid reservoir [19, 20].

$$\frac{P_{sys}}{\rho_f} + \frac{V_{sys}^2}{2} + gz_0 = \frac{P_1}{\rho_f} + \frac{V_1^2}{2} + gz_1 \quad (2)$$

The liquid reservoir is sized such that liquid level in the reservoir remains relatively constant despite any fluid volume changes in the cold plate or lift tube. By assuming that the fluid level remains constant and the system pressure is constant the fluid velocity in the upper reservoir is assumed to be small and Eq.2 can be simplified and re-arranged to determine the fluid pressure at the inlet of the cold plate in relation to the velocity. The fluid velocity,  $V_{sys}$ , at the inlet of the cold plate can be determined from the liquid volume flow rate of the system which will later be defined.

$$P_1 = P_{sys} + \rho_f gH - \rho_f \frac{V_1^2}{2} \quad (3)$$

where  $H = z_0 - z_1$

From mass conservation ( $\dot{m}_1 = \dot{m}_2$ ), an equation for the velocity of the two phase fluid leaving the cold plate can be derived. The cross-sectional area of the cold plate inlet and outlet will be assumed equivalent for this analysis.

$$V_2 = \frac{v_2 V_1}{v_f} \quad (4)$$

The specific volume of the fluid leaving the cold plate can be determined if the quality of the two phase fluid is known. The fluid quality is defined as the ratio of the fluid vapor mass to the total fluid mass.

$$x = \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_f} \quad (5)$$

$$v_2 = v_f + x(v_g - v_f) \quad (6)$$

Using Equations 4-5, the velocity of the fluid leaving the cold plate is determined as

$$V_2 = V_1 \left[ 1 + \left( \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_f} \right) \left( \frac{v_g - v_f}{v_f} \right) \right] \quad (7)$$

Assuming the density of the vapor to be negligible compared to the density of the liquid and assuming the specific volume of the liquid to be negligible compared to the specific volume of the vapor, Eq.7 can be simplified. Negligible vapor density corresponds to a negligible vapor mass flow rate which has been assumed.

$$V_2 = V_1 \left( 1 + \frac{\dot{V}_g}{\dot{V}_f} \right) \quad (8)$$

where  $V = \dot{m}v$

Assuming the vapor density to be negligible compared to the liquid density and the average velocity to be equivalent to the liquid velocity at the inlet of the cold plate, the momentum equation can be used to derive an expression for the fluid pressure of the two phase fluid leaving the cold plate.

$$P_2 = P_1 - \rho_f V_1 (V_2 - V_1) \quad (9)$$

Substituting Eq.3 and Eq.8 into Eq.9, the pressure of the fluid leaving the cold plate is described in terms of the vapor volumetric flow rate and the fluid velocity at the inlet of the cold plate as

$$P_2 = P_{sys} + \rho_f g H - \frac{\rho_f V_1^2}{2} - \frac{\rho_f V_1 \dot{V}_g}{A} \quad (10)$$

The conservation of momentum theory can be applied to develop another expression for the pressure of the fluid leaving the cold plate by relating the momentum entering the lift tube to the momentum at leaving the lift tube at the liquid vapor separator.

$$P_2 = P_{sys} + \frac{1}{2} f \rho_f V_2^2 \left( \frac{L}{A} \right) + \frac{W}{A} \quad (11)$$

The length of the lift tube (L) will be assumed to have a small effect on the bubble pump if the length remains short, less than two meters, but tall enough to house multiple expanding vapor slugs. For this analysis the maximum possible length of the tube, 0.2032m, was used. The maximum lift tube length was determined from the overall dimensional constraints given for the system and the dimensions and positions of the surrounding components. The weight of the liquid and vapor in the lift tube, W, can be determined from Newton's 2<sup>nd</sup> law of motion as

$$W = Lg(\rho_f A_f + \rho_g A_g) \quad (12)$$

Again assuming that the density of the vapor is negligible compared to the density of the liquid Eq. 12 becomes

$$W = Lg \frac{\rho_f A_f}{\left( 1 + \frac{\dot{V}_g}{s\dot{V}_f} \right)} \quad (13)$$

where  $\dot{V}_f = A_f V_f = AV_1$ ,  $\dot{V}_g = A_g V_g$ , and  $A = A_f + A_g$

In Eq.13 s is the slip factor describing the ratio of the vapor velocity to the liquid velocity. For slug flow the slip factor has been experimentally determined to be between 1.5 and 2.5 [21]. From an initial previous analysis it was determined that a lower slip factor corresponded to higher liquid flow rates. For this analysis a slip factor of 2.5 was assumed which will show the lowest analytical liquid flow rate results for the system operating in the slug flow regime.

Substituting Eq.8 and Eq.13 into Eq.11, the pressure of the fluid leaving the cold plate can be determined in terms of the liquid and vapor volume flow rates and the velocity of the fluid entering the cold plate as

$$P_2 = P_{sys} + \frac{4fL\rho_f V_1^2}{2D} \left( 1 + \frac{\dot{V}_g}{\dot{V}_f} \right) + g\rho_f \frac{L}{\left( 1 + \frac{\dot{V}_g}{s\dot{V}_f} \right)} \quad (14)$$

The friction coefficient K accounts for frictional losses in the lift tube. The coefficient can be determined from the friction factor which is determined by the Reynolds number. Assuming liquid laminar flow the Reynolds number, Re, friction factor, f, and friction coefficient, K, can be determined by the following equations.

$$Re = \frac{\rho_f V_1 D}{\mu_f}, \quad f = \frac{64}{Re}, \quad K = \frac{4fL}{D}$$

By setting Eq.14 equal to Eq.11 and using the definition of K from the above relation, the following relationship can be obtained for the lift tube

$$SR - \frac{1}{\left( 1 + \frac{\dot{V}_g}{s\dot{V}_f} \right)} = \frac{V_1^2}{2gL} \left[ (K + 1) + (K + 2) \frac{\dot{V}_g}{\dot{V}_f} \right] \quad (15)$$

Where SR is the submergence ratio (the ratio of the height of liquid in the reservoir to the fixed length of the lift tube is defined as  $SR = H/L$ ).

The liquid mass flow rate can then be determined for any combination of liquid height (H) and inner lift tube diameter (D) using

$$\dot{m}_f = \rho_f \dot{V}_f \quad (16)$$

### 3.1. Analysis Using Engineering Equation Solver (EES)

The liquid flow rate was determined for different diameters and submergence ratios using a parametric study with a diameter range of 1mm to 30mm and submergence ratio range of 0.1 to 0.9. Since the length or overall height of the lift tube (L) was set to be constant the submergence ratio was varied by changing the height of liquid in the reservoir (H). The liquid and vapor flow rates were used to determine the fluid quality. The effect of the lift tube diameter and the submergence ratio on the liquid flow rate and the quality are shown in Figs. 4 and 5, respectively. The results indicate that diameters smaller than 8mm would not generate any liquid flow due to large amounts of vapor which would occupy the nearly all of the lift tube corresponding to a transition from slug flow to annular flow. Also, it as it can be seen from Fig. 4, the liquid flow rate increased as the diameter increased. The increase in diameter corresponded to lower fluid qualities. Higher submergence ratios correspond to lower qualities and higher flow rates. This makes sense because larger diameters are capable of displacing more liquid volume per unit height due to the increase in the cross sectional area of the tube. However, the results show that lift tube diameters greater than 18 mm correspond to very low qualities which may not correspond to slug flow but rather bubbly flow. In the bubbly flow regime small bubbles rise up through the lift tube and are incapable of displacing liquid due to their size.

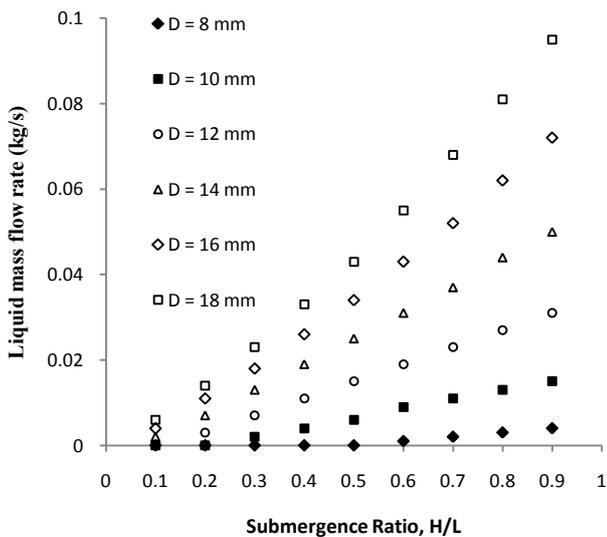


Fig.4. Liquid mass flow rate

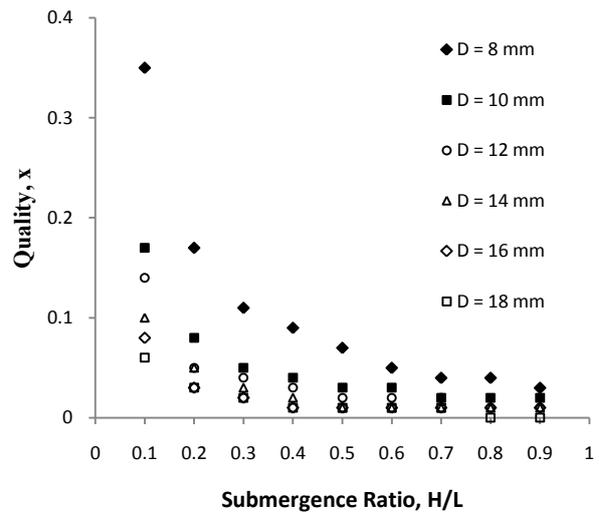


Fig.5. Fluid quality

To determine the appropriate liquid flow rate for the bubble pump cooling system a flow pattern chart was generated. A flow pattern chart displays the predicted transition boundaries between flow patterns. The experimentally and analytically determined lower line of the flow pattern chart shows the transition boundary between slug flow and bubbly flow Fair [22]. The upper line shows transition boundary between slug flow and annular flow. The upper boundary for this analysis was determined from earlier results predicting that annular flow was associated with lift tube diameters less than 8mm. In the bubbly flow regime small bubbles rise up through the lift tube and displace no liquid due to their size. In the annular flow regime a thin wall of liquid is formed around the tube wall and vapor moves through the center of the tube at high velocity. The flow pattern chart is shown in Fig. 6. In this figure, the mass velocity ( $m'$ ) is defined as

$$m' = \frac{\dot{m}_f + \dot{m}_g}{A} \quad (17)$$

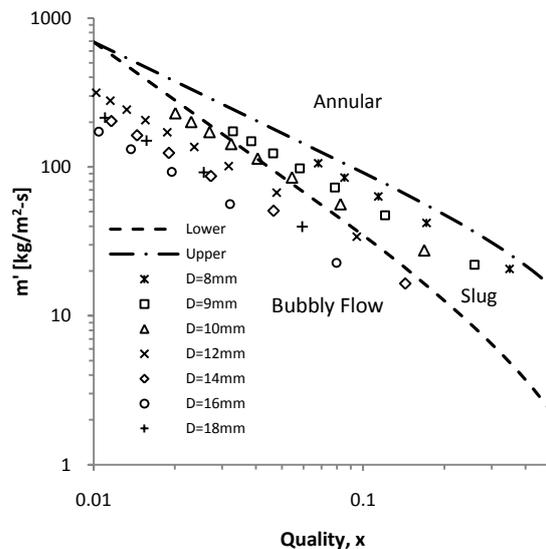


Fig.6. Flow Patterns

The area below the lower line on the flow pattern chart indicates the bubbly to single phase liquid flow regime. The area below the upper line and above the lower line indicates the slug flow regime. Finally, the area above the upper line indicates the annular to single phase vapor flow regime. From the flow pattern chart it was determined that the lift tube diameter must be between 8mm and 10mm. A lift tube diameter of 9.525mm (0.375") was selected because it is a standard size for high strength polycarbonate tubing and is inexpensive and readily available and is close in size to the optimal diameter of 9mm as predicted by the flow pattern chart.

The analysis was repeated with the 9.525mm diameter tube to determine liquid flow rate and fluid quality associated with different submergence ratios. It was determined that the submergence ratio between 0.3 and 0.6 would be applicable for the bubble pump cooling system. To determine optimum submergence ratio, the pressure gradient across the lift tube was determined by subtracting the pressure of the fluid entering the lift tube by the pressure of the fluid leaving the lift tube which is assumed to be near standard atmospheric pressure. The power required for an ideal pump to generate the flow was determined by multiplying the pressure gradient and the total volumetric flow rate for each of the applicable submergence ratios from

$$\dot{W}_{pump} = (\dot{V}_f + \dot{V}_g)(P_2 - P_{sys}) \quad (18)$$

Pump efficiency was determined by dividing the pump power by the total 400W of power, or heat rate, put into the system as

$$\eta = \frac{\dot{W}_{pump}}{Q} \quad (19)$$

Results from this analysis are shown in Table 1.

**Table 1. Pump power and efficiency**

Lift tube inside diameter, D = 9.525 mm			
SR	P <sub>2</sub> (kPa)	$\dot{W}_{pump}$	$\eta$ (%)
0.3	525	126.8	32
0.4	565	139.9	35
0.5	599.6	151.7	38
0.6	632.5	163.1	41

The submergence ratio of 0.6 is selected for the bubble pump cooling system design because it is associated with the highest efficiency.

#### 4. Testing and Sample Results

A prototype of the bubble pump cooling system was constructed and a picture of it is shown in Fig. 7. It was instrumented with thermocouples and pressure transducers. Two thermocouples, coated with thermal grease, were recessed into counter-bores that were machined in the copper lid near the surface of the cold plate. These thermocouples measure the surface temperature of the cold plate in two locations where the heaters are mounted. Thermocouples were also placed at the inlet and outlet of the bubble pump lift tube to monitor the temperature of the fluid entering and leaving the lift tube. A pressure transducer was installed in the cold plate to monitor fluid pressure. Another pressure transducer was installed in the

upper reservoir. The pressure transducers are used to monitor the system pressure and were used to determine the pressure drop across the lift tube. The pressure measurements were useful for ensuring that the pressure did not exceed 10 psig which is the maximum operating pressure of the silicon rubber hose used for plumbing the system. In addition, the pressure transducers are needed to monitor the pressure because if it becomes too high, that would impede the flow of methanol or in some cases prevent it completely. The wattage applied to the cold plate would be known by the current and voltage that were measured. A DasyLab program was set up to record the data from the P-Daq-65 unit.

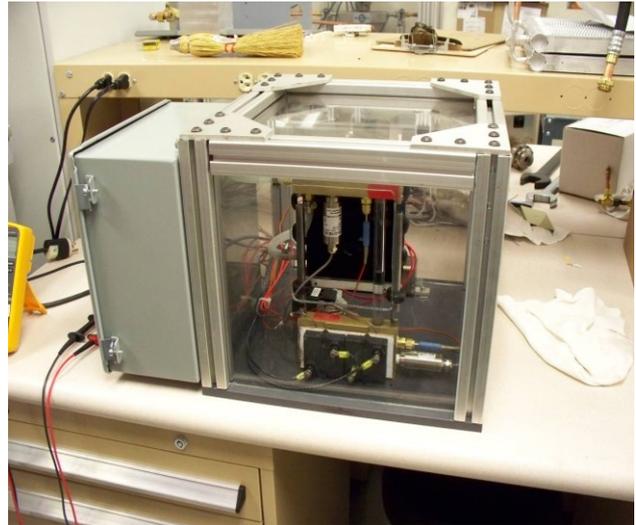


Fig.7. A picture of the prototype

Initial tests were carried out to verify the design for four different heating levels (100, 200, 300, 400 W) generated by the insulated gate bipolar transistor (IGBT). In these tests the height of lift tube of the bubble pump was 8 inches. For the heating level of 100 W, the system reached steady state with a cold plate surface temperature of 77°C (less than 80°C). However, for the other three heating levels, the cold plate surface temperature jumped above 80°C very quickly. In addition, the pressure increased as the temperature increased. As the pressure increases the temperature also increases and the flow is impeded. It was decided that the system needed more volume to allow for the fluid to expand and keep the system at a lower pressure. First the reservoir was replaced with a larger volume reservoir. The reservoir was mounted above the liquid level in the bubble pump so that the reservoir would not be flooded when the system was not running. It was observed that there was not much flow through the condenser so to solve that problem the 8 inches lift tube was replaced with a 24 inches lift tube. The longer bubble pump lift tube gave the fluid enough static head or momentum out of the upper reservoir to overcome the restrictions imposed by the condenser. The choice of 24 inches was made because an analysis was done to determine how much static head was needed to flow through the condenser and with the 24 inch tube there would be a sufficient amount of static head. The condenser was also mounted above the liquid level so that it too would not be flooded. Once these changes were made the system was ran again at heat loads of 100, 200, 300 and 400 Watts. Figs. 8 presents sample results for the case of 400 W heating load. As can be seen from the figure, the surface temperature of the cold plate remained below 80°C.

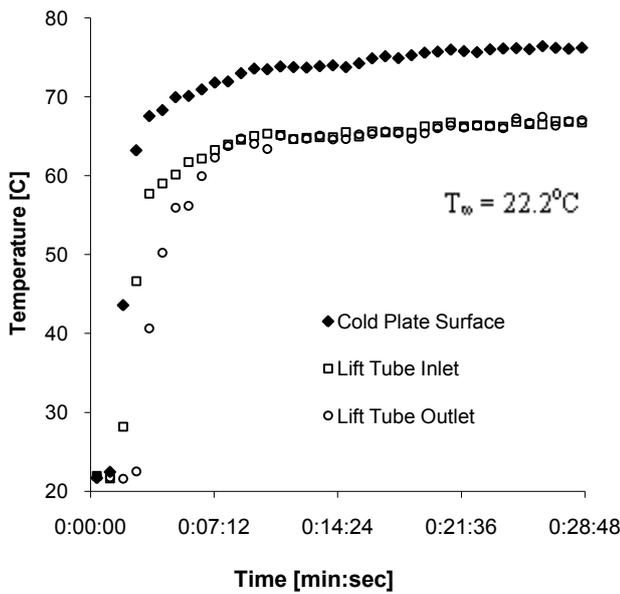


Fig.8. Performance of bubble pump for heating load of 400 W

### 5. Conclusion

A prototype of a bubble pump cooling system was designed, built and tested. Results indicate that it is feasible to build a bubble pump that will remove a certain amount of heat generated by an insulated gate bipolar transistor and keep its surface temperature under a certain value.

### Nomenclature

A	Cross-sectional area, m <sup>2</sup>
B	Perimeter of the lift tube, m
D	Diameter of the lift tube, m
f	Friction factor
g	Gravitational acceleration, m/s <sup>2</sup>
K	Friction coefficient
L	Length of the lift tube, m
$\dot{m}$	Mass flow rate, kg/s
P	Pressure, kPa
SR	Submergence ratio
T	Temperature of the jet, K
S	Slip between phases of two-phase flow
v	Specific volume, m <sup>3</sup> /kg
V	Velocity, m/s
$\dot{V}$	Volume flow rate, m <sup>3</sup> /s
x	Quality
Z	Elevation, m

### Greek Symbols

$\mu$	Dynamic viscosity, Pa.s
$\rho$	Mass density, kg/m <sup>3</sup>
$\sigma$	Surface tension

### Subscripts

0	State 0
1	State 1
2	State 2
f	Saturated liquid
g	Saturated vapor
max	Maximum
sys	System

### Non-dimensional Numbers

Re	Reynolds number
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### Acknowledgments

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