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Detailed Comparison of Natural Convection Heat Transfer between Solid Cylinders Placed Vertically and Horizontally in a Pool of Quiescent Air

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Abstract

The present paper addresses the behavior of heated solid cylinders at constant temperature that exchange heat with quiescent cold air by natural convection heat transfer. For the solid cylinders, two extreme orientations are examined in detail: one vertical and the other horizontal. A comparison between the vertical and horizontal orientations is carried out in terms of the three controlling parameters, namely two geometric parameters: the aspect ratio and the orientation (vertical and horizontal) in conjunction with one thermal parameter: the diameter-based Rayleigh number that is common to both orientations. The comparative natural convection problem is framed within the scope of heat transfer enhancement (or heat transfer suppression).

Keywords: *Natural Convection Heat Transfer; Solid Cylinders; Quiescent Air*

1. Preamble

The study of natural convection from heated solid cylinders to quiescent cold fluids is important in many engineering applications, like coils of HVAC systems, electronic components, wasted nuclear rods stored in fluid repositories, radiator space heaters and heat losses in human bodies, etc. On one hand, natural convection from a horizontal, heated solid cylinder at constant temperature to a still fluid has been investigated extensively. The solid cylinder placed in a horizontal position has one characteristic length, the diameter D. On the other hand, natural convection from a vertical, heated solid cylinder to a still fluid has not been investigated that broadly, but several articles have been published over the years. The solid cylinder at constant temperature when placed in a vertical position has two characteristic lengths: the diameter D and the height L. Surprisingly, there no information available in the specialized literature on convection heat transfer about the role of the two extreme orientations of solid cylinders with identical sizes (D and L) for enhancing heat transfer rates. Actually, the possible orientations for cooling solid cylinders are usually dictated by space requirements in the available air environment.

The topic of heat transfer enhancement has attracted interest to a multitude of industries worldwide from the standpoint of optimization. In past decades, an abundance of enhancing techniques has been devised for this endeavor [1]. The various techniques have been classified in three main groups:

(a) passive techniques absent of external power,
(b) active techniques, which require external power, and
(c) compound techniques, named for a suitable combinations of (a) and (b).

In the case of laminar natural convection from a vertical, heated solid cylinder, there are two possible pathways. When the boundary layer thickness δ is small compared to the diameter of the solid cylinder D, the mean Nusselt numbers may be determined by approximating the curved vertical surface as a flat vertical plate, i.e., ignoring effects of curvature. This simple condition is referred to as the “thick cylinder limit.” [2]. However, when the boundary layer thickness δ is large compared to the diameter of the solid cylinder D, effects of curvature must be taken into account. This constitutes a complex condition, which is referred to as “thin cylinder limit” [2]. Herein, the solution of the governing system of coupled conservation equations of mass, momentum and energy dispenses a plenitude of mean Nusselt numbers, which are jointly influenced by the diameter D and the height L.

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The first objective of the present work is to perform a detailed one-to-one comparison of natural convection heat transfer rates involving a long solid cylinder placed horizontally and an identical long solid cylinder placed vertically, when both are immersed in the same air environment. Thereafter, the second objective seeks to identify which of the two orientations of the solid cylinder brings forth heat transfer enhancement (or contrarily heat transfer suppression). In this regard, the parameters to be considered are the dimensions of the solid cylinder: diameter D and height L, the common diameter-based Rayleigh number, as well as the two extreme orientations: horizontal and vertical. To the author's knowledge, the one-to-one comparison of the two dissimilar heat transfer rates to be pursued in this work is not available in the specialized heat transfer literature.

2. Analysis

2.1. Vertical solid cylinders

Consider a heated vertical solid cylinder of diameter D and height L, constant surface temperature T_w and insulated caps immersed in quiescent air at a lower temperature as shown in Figure 1. The coupled system of conservation equations of mass, momentum, and energy for axi-symmetric, incompressible, laminar natural convection flow, together with the proper velocity and temperature boundary conditions was solved numerically by Day et al. [3] using ANSYS CFX 12.0, a finite-volume-based computational fluid dynamics code.

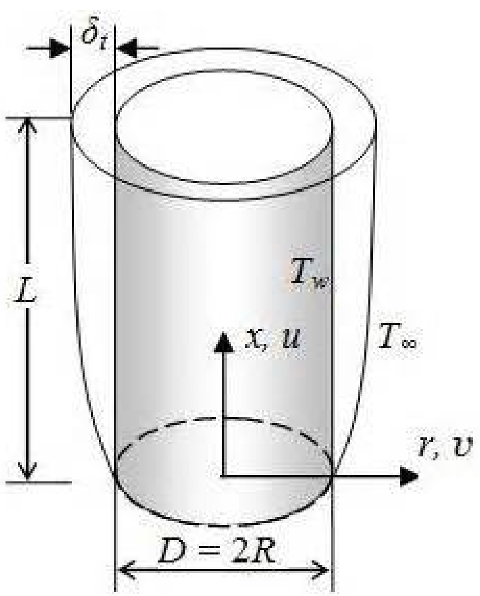

Figure 1. Schematic diagram of a vertical solid cylinder immersed in stagnant air

From the computed temperature fields *T(z, r),* the collected average wall heat flux $\bar{q}_w$ values were converted into associated mean convective heat transfer coefficients $\bar{h}$. This information was post-processed giving rise to the correlation equation for the height-based mean Nusselt number

$$\overline{Nu}_{L,V} = -\,0.062+0.544Ra_L^{1/4} + 0.61\left(\frac{L}{D}\right) \quad (1)$$

which applies to slender aspect ratios $2 \le \frac{L}{D} \le 10$, united to height-based Rayleigh numbers $10^2 \le Ra_L \le 10^9$ (laminar regime). Herein, the subscripts L and V in $\overline{Nu}_{L,V}$ indicate height and vertical position, respectively. A correlation equation (1) was developed by Day et al. [3] utilizing the MATLAB fitting tool along with the non-linear least squares method.

For purposes of homogeneity, it is convenient to re-express the pair of dimensionless groups $\overline{Nu}_{L,V}$ and Ra_L participating in eq. (1) in terms of the common diameter D and the aspect ratio $\frac{L}{D}$ of the solid cylinders. For this reason, $\overline{Nu}_{L,V}$ is multiplied by $\frac{D}{D}$, while Ra_L is multiplied by $\left(\frac{D}{D}\right)^{3/4}$ in eq. (1). Likewise, the operation leads to the alternate correlation equation for the diameter-based mean Nusselt number

$$\overline{Nu}_{D,V}\left(\frac{L}{D}\right) = -\,0.062+0.544\,Ra_D^{1/4}\left(\frac{L}{D}\right)^{3/4} + 0.612\left(\frac{L}{D}\right) \quad (2)$$

where the subscripts D and V in $\overline{Nu}_{D,V}$ indicate diameter and vertical position, respectively.

Vertical solid cylinders possessing three different aspect ratios $\frac{L}{D}$ satisfying the $\frac{L}{D}$ inequality $2 \le \frac{L}{D} \le 10$ will be examined in this work. First, for a large aspect ratio $\frac{L}{D} = 10$, the corresponding correlation equation (1a) turns into

$$\overline{Nu}_{D,V} = 0.606 + 0.306\,Ra_D^{1/4} \quad (3)$$

which is applicable to transformed diameter-based Rayleigh numbers contained in the Ra_D $\mathrm{sub-interval}$ $0.1 \le Ra_D \le 10^6$. Second, for an intermediate aspect ratio $\frac{L}{D} = 5$, (splitting the initial aspect ratio $\frac{L}{D} = 10$ in half), the corresponding correlation equation becomes

$$\overline{Nu}_{D,V} = 0.6 + 0.364\,Ra_D^{1/4} \quad (4)$$

which is applicable to transformed diameter-based Rayleigh numbers contained in in the Ra_D $\mathrm{sub-interval}$ $0.8 \le Ra_D \le 8\,x\,10^6$. Third, for a small aspect ratio $\frac{L}{D} = 2.5$ (splitting the aspect ratio $\frac{L}{D} = 5$ in half), the corresponding correlation equation (2) gets reduced to

$$\overline{Nu}_{D,V} = 0.587 + 0.433\,Ra_D^{1/4} \quad (5)$$

which is applicable to transformed diameter-based Rayleigh numbers contained in the Ra_D $\mathrm{sub-interval}$ $6.4 \le Ra_D \le 6.4\,x\,10^7$.

2.2. Horizontal solid cylinders

Switching next to a long horizontal solid cylinder of diameter D shown in Figure 2, the popular correlation equation for the diameter-based mean Nusselt number developed by Churchill and Chu [4] is

$$\overline{Nu}_{D,H} = 0.36 + 0.518\,\frac{Ra_D^{1/4}}{f(Pr)} \quad (6)$$

This equation depends on the diameter-based Rayleigh numbers and is restricted to laminar flows owning $\le 10^9$ and all Pr number fluids. Here, the subscripts D and V in $\overline{Nu}_{D,H}$ signify cylinder's diameter and vertical position, respectively. The

Prandtl number function $f(Pr)$ affecting the diameter-based mean Rayleigh number Ra_D in eq. (6) is expressed by

$$f(Pr) = \left[1 + \left(\frac{0.56}{Pr}\right)^{9/16}\right]^{4/9} \qquad (7)$$

When the working fluid is air with $\mathrm{Pr} = 0.7$, then $f(Pr) = 1.325$ and henceforth the correlation eq. (6) is simplified to

$$\overline{Nu}_{D,H} = 0.36 + 0.391\, Ra_D^{1/4} \qquad (8)$$

alongside the same limitation $Ra_D \leq 10^9$.

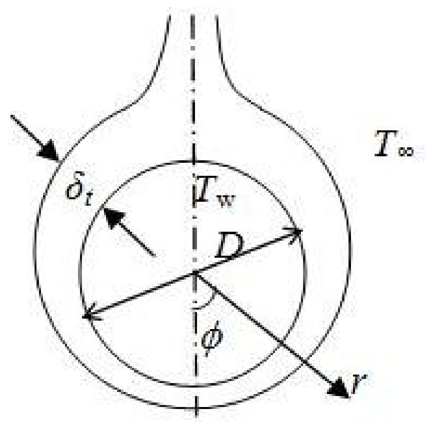

Figure 2. Schematic diagram of a horizontal solid cylinder immersed in stagnant air

3. Numerical Results

To assess the influence of the two different orientations of the heated solid cylinder, either vertical or horizontal, both immersed in an air natural convection environment, the two prevalent mean convection coefficient ratios $\frac{\bar{h}_{D,H}}{\bar{h}_{D,V}}$ and $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}}$ will be employed.

First, for a large aspect ratio $\frac{L}{D} = 10$ permissible in eq. (1), the maximum heat transfer enhancement occurs in the high Ra_D sub-region ($Ra_D \leq 10^6$), where the vertical orientation of the solid cylinder is favorable. In reference to the third column in Table 1, it is found that the mean convection coefficient ratio $\frac{\bar{h}_{D,H}}{\bar{h}_{D,V}} = 1.24$ (24% better), at the highest $Ra_D = 10^6$, decreasing to $\frac{\bar{h}_{D,H}}{\bar{h}_{D,V}} = 1.10$ (10% better) at an intermediate $Ra_D = 10^3$. Contrarily, moving next to the lower Ra_D sub-region ($0.1 < Ra_D < 10$), the horizontal orientation of the solid cylinder is favorable. Further, at a low $Ra_D = 10$, the mean convection coefficient ratio $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}} = 1.09$ (9% better), growing to a maximum $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}} = 1.22$ (22% better) when Ra_D descends to a lower value of 1.

For an intermediate aspect ratio $\frac{L}{D} = 5$, the maximum heat transfer enhancement happens in the high Ra_D sub-region ($Ra_D \leq 8\,x\,10^6$), where the vertical orientation of the solid cylinder is beneficial. That is, resorting to the third column in Table 2, the mean convection coefficient ratio $\frac{\bar{h}_{D,H}}{\bar{h}_{D,V}} = 1.06$ (6% superior), at the highest $Ra_D = 8\,x\,10^6$, decreasing to an imperceptible mean convection coefficient ratio $\frac{\bar{h}_{D,H}}{\bar{h}_{D,V}} = 1.01$ (1% superior) at a moderate $Ra_D = 10^3$. Conversely, for the lower Ra_D sub-region ($Ra_D > 0.8$), the horizontal orientation of the solid cylinder is beneficial. In fact, at a very low $Ra_D = 10$, the mean convection coefficient ratio $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}} = 1.18$ (18% superior), growing to a maximum of $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}} = 1.22$ (22% superior) when $Ra_D = 1$.

For the small aspect ratio $\frac{L}{D} = 2.5$ allowable in eq. (1), which is linked to the Ra_D interval in $6.4 \leq Ra_D \leq 6.4\,x\,10^7$, the fourth column in Table 3 indicates that the vertical orientation for the solid cylinder is always advantageous over the opposing horizontal orientation, regardless of the Ra_D value. This variability implies that all mean convection coefficient ratios $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}} > 1$. In numbers, the minimum heat transfer enhancement $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}} = 1.11$ (11% improvement) takes place at the largest $Ra_D = 6.4\,x\,10^7$. Moreover, it is also observable that the mean convection coefficient ratio $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}}$ increases continually with decrements in Ra_D, until reaching the maximum heat transfer enhancement $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}} = 1.29$ (29% improvement) at a small $Ra_D = 10$. This Ra_D is close to the smallest $Ra_D = 6.4$ in the Ra_D sub − interval.

Table 1. Comparison between vertical and horizontal solid cylinders with large aspect ratio $\frac{L}{D} = 10$ engaging the Ra_D interval: $0.1 \leq Ra_D \leq 10^6$

Ra_D	$\overline{Nu}_{D,H}$	$\overline{Nu}_{D,V}$	$\frac{\bar{h}_{D,H}}{\bar{h}_{D,V}}$ $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}}$
10^6	12.725	10.283	1.24
10^3	2.559	2.327	1.10
10	1.055	1.150	1.09
1	0.751	0.912	1.22

Table 2. Comparison between vertical and horizontal solid cylinders with intermediate aspect ratio $\frac{L}{D} = 5$ engaging the Ra_D interval: $0.8 \leq Ra_D \leq 8\,x\,10^6$

Ra_D	$\overline{Nu}_{D,H}$	$\overline{Nu}_{D,V}$	$\frac{\bar{h}_{D,H}}{\bar{h}_{D,V}}$ $\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}}$
$8\,x\,10^6$	21.155	19.960	1.06
$4\,x\,10^3$	3.470	3.495	1.01
10	1.055	1.247	1.18
1	0.751	0.912	1.22

Table 3. Comparison between vertical and horizontal solid cylinders with small aspect ratio $\frac{L}{D} = 2.5$ engaging the Ra_D interval: $6.4 \leq Ra_D \leq 6.4\,x\,10^7$

Ra_D	$\overline{Nu}_{D,H}$	$\overline{Nu}_{D,V}$	$\frac{\bar{h}_{D,V}}{\bar{h}_{D,H}}$
$6.4\,x\,10^7$	39.316	35.332	1.11
$3.2\,x\,10^7$	33.150	29.768	1.11
10^2	1.893	1.596	1.18
10	1.357	1.055	1.29

In order to determine the commensurate $\overline{Nu}_D$ inflection points for the three selected aspect ratios $\frac{L}{D} = 10, 5, 2.5$ in the corresponding Ra_D sub − intervals cited before, the trio of equations, consisting in eq. (3) for $\frac{L}{D} = 10$, eq. (4) for $\frac{L}{D} = 5$, and eq. (5) for $\frac{L}{D} = 2.5$ must be paired with eq. (8). First, for the large aspect ratio $\frac{L}{D} = 10$, the $\overline{Nu}_D$ inflection point associated with eq. (3) turns out to be a diminutive $Ra_D = 70.15$. Second, for the intermediate aspect ratio $\frac{L}{D} = 5$, the $\overline{Nu}_D$ inflection point connected to eq. (4) shifts to a moderate $Ra_D = 6\ x\ 10^3$, i.e., about two orders of magnitude higher than the previous $Ra_D = 70.15$ related to $\frac{L}{D} = 10$. Third, it is striking to realize that there is no $\overline{Nu}_D$ inflection point for the small aspect ratio $\frac{L}{D} = 2.5$ chosen in this work within the bounds of eq. (1). As a point of reference, it is worth stressing that the lowest aspect ratio connected to the correlation equation (1) is a chubby $\frac{L}{D} = 2$, i.e., a value close to 2.5.

4. Conclusions

The following conclusions may be drawn for the trends exhibited by the natural convection heat transfer from solid cylinders placed horizontally and vertically, both submerged in quiet air at a lower temperature. The horizontal orientation outperforms the vertical orientation by a remarkable margin of up to 24% under combinations of moderate-to-large aspect ratios with $\frac{L}{D} = 10$ and 5 with large diameter-based Rayleigh numbers Ra_D. Conversely, the opposite situation dealing with the horizontal orientation is preferable for other combinations of moderate-to-large aspect ratios with $\frac{L}{D} = 10$ and 5 and small diameter-based Rayleigh numbers Ra_D. This results in a generous margin of up to 22%. A notable exception lies in the small aspect ratio $\frac{L}{D} = 2.5$, wherein the vertical orientation is worthier than the horizontal orientation for all applicable diameter-based Rayleigh numbers Ra_D. Actually in this limiting case, the dominance level ranges between a somewhat discreet 11% factor and a realistic gigantic 29% factor.

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