Numerical Study of Heat and Mass Transfer during the Thermal Drying of Tropical Woods

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Abstract
A sector of utilization and transformation of wood give much money at the countries of the central Africa region. If we want to increase these advantages, it is important to do locally the first and second transformations of wood. Self-control of the wood drying is necessary to ameliorate a wood quality and to preserve an art work doing in wood. In this article, we are modeling a drying of one piece of bete wood (*Mansonia altissima*) with dimensions are 1m of length and thickness is no more than 25cm. We have used a literature to obtain a mass and heat equations and the thermophysical properties of the present wood. We have considered some thermophysical properties that we are unkempt in the precedent work [1]. We have experimentally obtained a relationship between a density of this wood with a water content. The others thermophysical properties come from the literature [1,2]. We have simulated a wood drying in the constant conditions and in the conditions of the bete drying table established by CIRAD Organization. A sensibility study is doing to validate a modeling. Then, the present modeling explains temperature, mass fraction of the vapor in air and water content evolutions during the industrial process of drying. This modeling can to be used for to overhaul a drying table of bete wood and the others of tropical woods. This work uses a gaseous pressure can be used to explain the drying with the high temperatures conditions.

Keywords: Drying- Heat and Mass Transfer -Forced Convection-Modeling- Finite Difference- *Mansonia Altissima*

1. Introduction
Tropical countries have vast forests that contain many species of wood [3-5]. These varieties enable the majority of the principal European and Asian importing these woods to be very interested. Unfortunately, the majority of wood is exported from Africa in rough timber, as many African countries as Cameroon and Gabon require that the first transformation must be done locally [6]. In effect, local transformation of wood enables job creation, promotes employment relative to the transformation of the wood and to struggle against deterioration of the exchange terms because, when the wood is sold in Africa, there are to be comes very expensive. Though, economies of the tropical countries do independ of the exterior market and the effects as the recent worldly financial crisis. Cameroon has many forests and surface mine potentials to become one of the rich African countries [4]. Then it is very important to develop a technology and science of tropical countries in order to answer the immediate needs as preservation of wood by thermal drying. Thus, the utilization of the forest is optimized and ecological advantages as fixation of carbon, regeneration of oxygen, preservation of animal diversity, of grass and trees can be preserved. In this paper, we applied one model which explains the drying of tropical woods [1] to describe the drying the tropical woods applied on the bete (*Mansonia altissima*). This wood is abundant in the region of Central Africa and is highly demanded in the interior and exterior markets of Cameroon [7]. In the previous work [1], we are supposed that heat and mass transfer air are constant. In the present work, we take a real variation of all thermophysical parameters indicate in all equations of transfer with integration of air pressure. This supposition traduces a real behavior of industrial drying and can be used to ameliorate the drying tables of tropical woods and consequently, to ameliorate the quality of dried wood.

2. Modeling and Thermophysical Parameters
A present model is developed in the literature and has been validated with a numerical simulation of the drying of ayous and ebony woods in others conditions [1]. It is very difficult to use all drying models without simplifications because a drying process shows multidimensional characteristics and the
samples to dry have many thermophysical parameters unknown [8]. Therefore, we are taking in this work the following hypothesis:

- A size of sample to dry is constant, homogeneous and it is chemically inert;
- As soon as the drying begins, water that comes from wood is made of vapor and free water, bound water is extracted when the fiber saturation point is obtained;
- We have a symmetric drying, therefore a median of the wood plank is an extreme temperature and water content;
- A heat transfer by convection takes place only at the level of thermal layer limit who to be at the borders of samples;
- We have neglected the transfer on the lateral faces. Transfers which take place are controlled by the thickness of the planks, which is smaller than width and length;
- Gravity effect and hydraulic conductivity of wood are neglected;
- The losses inherent in the dryer are not respected. All the other values are supposed to be transferred to the wood;
- Air and water are supposed to be incompressible.

We obtained the following equations (1) below [1]:

\[
\frac{\partial H}{\partial t} - \nabla \cdot (\overline{D_m} \nabla H + \overline{D_r} \nabla T) = 0 \tag{1a}
\]

\[
\rho_s C_v \frac{\partial T}{\partial t} - \nabla \cdot (\overline{D_r} \nabla T + \overline{D_m} \nabla H) = \nabla \cdot \vec{\lambda} \nabla T \tag{1b}
\]

Between wood and air, we have:

\[
\overline{\lambda} \nabla T = \overline{h_r} (T - T_{air}) n + \rho_s L D_{diff} \nabla H \tag{1c}
\]

\[
- \overline{D_m} \nabla H = \overline{h_0} (H - X_{eq}) n \tag{1d}
\]

with:

- H: Water content of wood (kg/kg)
- T: Temperature of wood (K)
- t: Drying time (s)

\( \overline{h}_r, \overline{h}_m \): Global coefficient transfer respectively of the heat (W/(m².K)) and the mass (m/s)

\( \rho_s \): Density of the anhydride wood (kg/m³)

\( X_{eq} \): Equilibrium water content (kg/kg)

\( C_v \): Specific heat capacity of wet wood (J/(kg.K))

\( \lambda \): Thermal conductivity of wet wood (W/(m.K))

At the median plane, the fluxes diffusion of humidity and heat are neglected.

Diffusions coefficients are such as:

\[
\overline{D_m} = \overline{D_r} = \frac{\rho_s k_s \overline{h}_r}{\rho_s} \frac{\partial P}{\partial H} = \frac{\rho_s L D_{diff}}{(1-C) \partial H} \frac{\partial C}{\partial H} \tag{2}
\]

\[
\overline{D_m} = \overline{D_r} = \frac{\rho_s k_s \overline{h}_r}{\rho_s} \frac{\partial P}{\partial H} = \frac{\rho_s L D_{diff}}{(1-C) \partial H} \frac{\partial C}{\partial H} \tag{3}
\]

Elements of second member of equations (2) and (3) indicate in the order the contributions of the coefficients to extract bound water, free water and the vapor of the water.

\[
\overline{D_r} = \frac{(E + L) \rho_s D_r \frac{\partial C}{\partial T} - E \rho_r k_s \overline{h}_r \frac{k_r}{\rho_r} \frac{\partial P}{\partial T}}{1 - C} \tag{4}
\]

\[
\overline{D_m} = \frac{(E + L) \rho_s D_m \frac{\partial C}{\partial H} - E \rho_r k_s \overline{h}_r \frac{k_r}{\rho_r} \frac{\partial P}{\partial H}}{1 - C} \tag{5}
\]

Elements of second member of equations (2) and (3) indicate in the order the contributions of the energies to extract water in the state of vapor and in the state of liquid.

With: \( D_r \), \( D_m \) : diffusion coefficients of bound water respectively at a gradient humidity (m²/s) or at a gradient temperature (m²/(Ks)) ;

- E: heat desorption of the water absorbed (J/kg) ;
- L: latent heat of the vaporization of liquid water (J/kg) ;
- \( \alpha \) : thermomigration coefficient (K⁻¹).

In the space order, terms of second member of the equations (4) and (5) show the energies contributions to extract water in vapor state or liquid state.

*Thermal conductivity of wood (W/(mK)) [2]:

\[
\lambda = \frac{P_k}{\rho_r} (0.2003 + 0.00548H) + 0.02378 \tag{6}
\]

H in %

*Specific heat capacity of wood (kJ/(kg.K)) [2]:

\[
C_v = \frac{C_{Po} + 0.01HC_{pe}}{1 + 0.01H} + H(-0.6191 + 2.36*10^{-4}T - 1.33*10^{-4}H) \tag{7a}
\]

H in %

\( C_{Po}=0.1031+0.003867T \quad \text{and} \quad C_{pe}=4.19kJ/(kg.K) \) (7b)

T in Kelvin (K).Second term of second member of (7a) is equal to zero in a non hygroscopic domain, domain that is limit by the water content at the fiber saturation points (FSP).

*Fiber saturation points

We are experimental obtained this parameter after determination the water content of the wood where beginning the first variations of volume in each temperature. We have obtained the following equation :

\[
H_s=0.8397-0.0018T \quad \text{avec} \quad R^2=0.9643 \text{et} \ T_s \text{en K} \tag{8}
\]

*Latent heat of vaporization (J/kg)[9] :

\[
L=(3335-2.91T)10^3 \tag{9}
\]

L in J/kg, H in % and T in K

*Desorption heat of the water adsorbed (J/kg) [10] :

\[
E=1170.4x10^3 \exp(-0.14H) \tag{10}
\]

E in J/kg and H in %.

*Coefficient of diffusion of the vapor of water in the air (m²/s) [11]:
\[ D_g = 2.25 \times 10^{-4} \left( \frac{T_0}{273} \right)^{1.84} \]  

(11)

\( P_g \) is the pressure of air estimated at 10^5 Pa.

* Coefficient of diffusion of the vapor of water in the wood (m^2/s) [12]:

\[ D_g = fD_G \]  

(12)

With \( f \) is brake factor which retain the diffusion of the vapor of water in the wood. We are taking the value 10^{-5} [12].

* Coefficient of diffusion of the bound water in the wood (m^2/s) in the dimension 1 [13]:

\[ D_w = \exp \left[ -9.9 + 9.3X_{aq} \right] \frac{4300}{T} \]  

T in K

(13)

*Coefficient of diffusion of the water in the wood caused to the gradient of temperature (m^2/(K.s)) in the dimension 1 [14]:

\[ D_w = \frac{dX_{aq}}{dT} \]  

(14)

*Wood permeability [15]:

\[ k_w = k_a \cdot 5 \times 10^{-6} \text{ m}^2 \]  

(15)

*Capillary pressure (Pa) [15]:

\[ P_c = \left( 2.03 \times 10^{-5} \cdot \exp(-3.78c) + 9.1 \cdot c^{-44} \right) \left( 1 - 2.19 \times 10^{-4} (T) \right) \]  

(16a)

\( c^* \) is the saturation of wood by the vapor and T in °C.

\[ \zeta = 1 - \frac{P_o}{15000} \]  

(16b)

*Density of the water (kg/m^3) [16]:

\[ \rho_w = -0.0039T^3 - 0.0557T^2 + 1002.6 \]  

(17)

*Activation energy of the wood (J/mol) [17]:

\[ E_a = 4.18 \left( 9200 - 7000X_{aq} \right) \]  

(18)

*Density of the vapor of water (kg/m^3) [16]:

\[ \rho_{aq} = \frac{353}{T_o} \]  

T in K

(19)

*mass fraction of the vapor [12]

\[ C = \frac{HR \cdot P_{sat}}{0.622 (P_g - HR \cdot P_{sat})} \]  

(20)

*Air humidity [12]:

\[ y = \frac{0.622 \cdot HR \cdot P_{sat}}{P_g - HR \cdot P_{sat}} \]  

(21)

*Air humidity of the air at the saturation state (%)[17]:

\[ y_{aq} = \frac{10^4}{8.32(T_o + 273)} \exp \left[ 14.02 - \frac{5215}{T_o + 273} \right] \text{ T_o in °C} \]  

(22)

*Pressure of the water in the saturation state (Pa) [18]:

\[ P_{sat} = \exp \left( 25.5058 - \frac{5204.9}{T_o} \right) \text{ T_o in K} \]  

(23)

*Soret effect coefficient (K^-1) [14]:

\[ \alpha = \frac{dX_{aq}}{dT} = \frac{E_{HR}}{RT^2} \frac{\partial X_{aq}}{\partial HR} \]  

(24)

*Energy of the activation of wood (J/mol) [19]:

\[ E_a = 4.18 \left( 9200 - 7000X_{aq} \right) \]  

(25)

*Equilibrium water content (kg/kg) [2]:

\[ X_{aq} = \frac{k_{HR}}{w} \left( 1 - k_{HR} + k_{HR} \cdot k_{a} \cdot HR \right) \]  

(26a)

\[ w = 349 + 1.29T_o + 0.0135T_o^2 \]  

(26b)

\[ k_a = 0.805 + 0.000736T_o - 0.00000273T_o^2 \]  

(26c)

\[ k_H = 6.27 - 0.00938T_o - 0.0000303T_o^2 \]  

(26d)

\[ k_{HR} = 1.91 + 0.047T_o - 0.000293T_o^2 \]  

(26e)

\[ T_o \text{ in °C} \]

*Prandtl number [16]

\[ Pr = 2.54 \times 10^{-5} T_o + 0.7147 \]  

(27)

*Thermal conductivity of the air (W/(m.K)) [16]

\[ \lambda_o = 5.75 \times 10^{-5} T_o + 0.0242 \]  

(28)

*Dynamical viscosity of the air (Pa.s) [16]:

\[ \mu_o = 10^{-5} \left( 0.0046 T_o^2 + 1.7176 \right) \]  

(29)

*Dynamical viscosity of the water (Pa.s) [16]:

\[ \mu_w = 10^{-4} \left( 0.002 T_o^2 - 0.3389 T_o + 1.1999 \right) \]  

(30)

*Mass transfer coefficient (m/s) [15]:

\[ h_m = 9.454 \times 10^{-3} V_o^{0.3003} \]  

(31)

*Heat transfer coefficient (W/(m^2.K)) [15]:

\[ h_v = 1.328 \frac{k_v}{L_{po}} \text{ Re}^{1.2} \text{ Pr}^{1.5} \]  

(32)

*Density of wet wood:

We have experimentally determined this parameter. We have obtained:

\[ \rho_w = 529.08 H + 617.81 \]  

(33)

\( \rho_w \) in kg/m^3

With H in decimal and relative incertitude equal to 0.931%.

*Expressions of all derivatives

Partial derivatives of the capillary pressure and the mass fraction of the vapor are being able to aggravate experimental noise. We have used the following equations:

\[ \frac{\partial C}{\partial T} = \frac{C(T_{bois}) - C(T_{air})}{T_{bois} - T_{air}} \]  

(34)

\[ \frac{\partial C}{\partial H} = \frac{C(T_{bois}) - C(T_{air})}{y(T_{bois}) - y(T_{air})} \]  

(35)

\[ \frac{\partial P}{\partial T} = \frac{P(T_{bois}) - P(T_{air})}{T_{bois} - T_{air}} \]  

(36)

\[ \frac{\partial P}{\partial H} = \frac{P(T_{bois}) - P(T_{air})}{y(T_{bois}) - y(T_{air})} \]  

(37)
3. Numerical Method of the resolution

To find numerical resolution of the analytical equations obtained, we have utilized the finite difference method. This method allows to approximate the values of continued functions and continually derivable by the development of Taylor’s series. Implicit form is adapted at the resolution of differential equations less strongly coupled and to have the coefficients that less change. Coupling of these differential equations cannot envisage an analytical resolution. Wood plank is supposed to be a thick layer divided a 2N+2 series of thin layers, with in a half thickness the spaces orders j goes to 1 at N+1. Forms of mass and heat discrete equations are given by:

\[
\begin{align*}
-\rho C_{p,j} \frac{\partial T_j}{\partial t} + \frac{\partial}{\partial x}(D \frac{\partial T_j}{\partial x}) &= A_T [T_{j-1} - T_j] + (1 + 2\alpha) T_j - AH'_{j+1} - \rho C_{p,j} \frac{\partial x}{\partial t} \\
BT_j &+ (2 + 1) T_{j+1} - BT_{j+1} = T_j - 2CH'_{j+1} + CH'_{j+1}
\end{align*}
\]

(38.a)

With:

\[
A_T = \frac{D_T \rho C_{p,j} T_j}{(\Delta x)^2}
\]

(38.b)

A, B, C and D are given at the knots (i,j) with:

\[
x_j = x_i + jh, \quad (i,j) = (i,0)
\]

(39)

We have put the system (21.a) in the following matrix form:

\[
\begin{bmatrix}
[E_j] G'_{j+1} + [F_j] G'_{j+1} + G'^{\prime}_j &= [H_j] \\
F_j G'_{j+1}
\end{bmatrix}
\]

(40.a)

With:

\[
G'_j = \begin{bmatrix}
H'_j \\
T'_j
\end{bmatrix}, \quad [E_j] = \begin{bmatrix}
2A+1 & 0 \\
0 & 2B+1
\end{bmatrix}, \quad [F_j] = \begin{bmatrix}
-A \\
0
\end{bmatrix}
\]

(40.b)

In order to have a recursive solution, we have considered that:

\[
G'^{\prime}_j = [\beta_j] - [F_j] G'^{\prime}_{j+1}
\]

(41.a)

With:

\[
[F_j] = \begin{bmatrix}
[\beta_j] & [F_j] G'^{\prime}_{j+1}
\end{bmatrix}
\]

(41.b)

Results can be progressively obtained. We have translated our program in the class 77 of fortran to generate all numerical results. Excel permitted us to draw our curves. The time step and space step are respectively 1700s and 6.25x10^{-3}m. Initial temperature of samples is 28°C. In each drying time, from local values of humidity and the temperature of wood, average values are evaluated through relations 42.a and 42.b.

\[
T_i = \frac{\sum T_{i-1}}{\Delta x}
\]

(42.a)

\[
H_i = \frac{\sum H_{i-1}}{\Delta x}
\]

(42.b)

These are the values that will be represented on the following figures.

4. Results and Discussion

4.1. Experimental validation of the kinetic of drying applied on bete with constant air characteristics

We are obtained experimental water content in each drying time respective then to validate the present modeling. We are used constant characteristics of air as the experiment. These characteristics are: Tair= 60°C, , relative humidity of air equal to 30% and thickness of the plank is 25mm. According to figure 1, we note a same behavior between theoretical and experimental points. Here, we are determined in inverse mass transfer coefficient with heat transfer fixed at 11.2W/m²°C [1]. We are obtained h_m=2.5x10^{-3}m/s and a= 0.0045K^{-1}.

4.2. Study of the sensibilities of our modeling during the drying of bete wood

The plank of wood that we dried numerically have 1m of length and the thickness will be precised each time. The drying is symmetric and the air circulates in the sense of the plank. All relationships on the paragraph 2 are utilized. The flow regime is laminar because Reynolds number is less than 3*10^{4}. Drying table proposed by the Centre de Coopération International en Recherche Agronomique pour le Développement (CIRAD) [20] is utilized in the following. This drying table is illustrated in the table 1 for the thickness c<38mm:

<table>
<thead>
<tr>
<th>Humidity of wood (%)</th>
<th>Temperature (°C)</th>
<th>Relative Humidity of the air (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>green</td>
<td>50</td>
<td>47</td>
</tr>
<tr>
<td>40</td>
<td>50</td>
<td>45</td>
</tr>
<tr>
<td>30</td>
<td>55</td>
<td>47</td>
</tr>
<tr>
<td>20</td>
<td>70</td>
<td>55</td>
</tr>
<tr>
<td>15</td>
<td>75</td>
<td>58</td>
</tr>
</tbody>
</table>

Figure 2 presents evolutions of the water content and the temperature of the wood in function of the drying time and the thickness of the plank. We note that drying time and thickness increase together when kinetic of the air and initial water content are constant. In effect, an explanation is the fact that when thickness increases, water in the wood is much confined. For to pass at the water content 60% to 20%, thickness of the plank of wood is 15mm, 20mm or 25mm, the drying time required is respectively 31.5h, 52.5h and 79h, with the kinetic of the air is equal to 1.5m/s and the initial water content is 60%. This same figure shows that when a thickness of the plank increases. In effect, the time of the repartition of the temperature in the thickness of plank become important, the energetic consummation is much important when the mass of wood to heat up increases. At the end of the drying, water content is not function of the thickness of the plank because water content at the equilibrium is not function of the temperature.
Figure 3 shows that it is costly to dry the plank initially most humid because, more the wood is humid, less rapidly we attained the characteristics of the utilization of the plank. This same figure indicates that equilibrium water content is not function of the initial water content.

Figure 4 shows that when the speed of the circulation of air increases, the speed of evolution of temperature of the wood increases equally, but the influence of this parameter is small at the level of evolution of water content. In this thickness, a drying is most influenced by interior migration of humidity. An important value of the speed of air causes a superficial drying of the plank than the interior layers. This surface can alternatively to dry and to absorb humidity and by consequently to destroy cellular tissues and the quality of the wood dried. High speeds can also to create turbulence and consequently it is possible to disturb the control of the characteristics of drying air through the plank.

Figure 5 presents the evolutions in the time of mass fraction of the vapor of water in air, of humidity of air and of humidity of air at the saturation state. With the time, air absorbs humidity given by wood. It is necessary, for to increase the drying kinetic (and to reduce the drying time), to evacuate the air of the dryer before his saturation. We note that the humidity of air in the dryer is nearly saturated after 55h of the drying. In the following of the drying, the reduction of relative humidity of the air and the increasing of the dry temperature permitted to increase the power of evaporation of the air.

On the figures 2, 3 and 4, we can to observe the quickly variations of the temperature of the wood, of the air humidity and the mass fraction of the vapor in the air which are caused by abrupt changes of the characteristics of the drying air in order to respect the drying table.
Increase in the temperature of air permits to create the
gradients of temperature in the plank and to generate the
gradients of humidity which explain the movement of the water
in the wood. It is important to control these gradients in other
to reduce the barrier to the extraction of water and to favour an
important not homogeneity of the humidity in the thickness of
the plank. Important gradients can to create many failures as
collapses and dimensions.

Figure 7 shows that during the drying of wood, the water
content on the surface of wood is equal to the value at the
equilibrium. We note that in general, we cannot to rich at the
equilibrium point because water content of the utilization is
important. Different abrupt modifications on the water content
observed in the figure 7 are caused by the different modifications of the drying table. These modifications permit
to reduce the equilibrium water content of water.

It is most important to determine the own thermophysical characteristics of each wood studied because, a bad estimation
can to generate a bad estimation of the energy required for to
dry well our product. For example, we have evaluated the
influence of liquid permeability on the wood. We observed that
the variation of permeability shows a light variation of the
kinetic (figures 8a and 8b). A good vision of the graph after
60h of the drying shows that a bad estimation of the permeability can to change the value of final water content
(figure 8b). In according to the literature, we note a small influence of the liquid permeability of the porous material on
the drying kinetic [21].

5. CONCLUSION

For to dry biological material well good as bete wood, it is
important to do the prevision on the requisite energy and the
behavior of the material to dry when quality is necessary.
Present modeling is a modest contribution to study the drying
of tropical wood applied here on bete wood. Digital simulation
obtained give general idea on evolutions with the drying time
of the temperature, water content, humidity of air and the
fraction of vapor in the drying air. All evolutions give a
satisfaction behavior according to physic explanations. As
example, a less influence of the permeability of the wood on
the drying kinetic and absorption of humidity in the dryer by
the drying air. This modeling can to be used to simulate the
drying of biological materials at the high temperatures. As
perspectives, for to ameliorate ours results, it is important to
characterize experimentally the thermophysical parameters of the
wood studied in function of temperature, thickness of material,
relative humidity and the kinetic of drying air because many
researchers said that thermophysical parameters varied with
wood species and anatomical direction [22-34]. A great
number of samples are necessary to generalize results.

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